ANALYSIS OF CIRCULAR PLATE WITH TUNABLE FREQUENCY

BY

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ABSTRACT

The study of plate with tunable frequency becomes an important study as it aids for more effective design of plate for the use of devices such as resonator (i.e. SJAs, speakers) or energy harvesting devices. This study explored on idea of a structure which its frequency may be tuned through the application of electrical means alone. This idea would eliminate the needs of extra mechanical structure in certain transportation such as aircraft. A circular plate is tuned through attaching a piezoceramic annular plate at the edge of a circular plate. By controlling the voltage applied to the piezoceramic annular plate, thus the radial load, the fundamental frequency of the circular plate may be tuned. The analysis is performed by using finite different method which coded through MATLAB®. The study is divided into two major parts. First part is the study of circular plate buckling problem in order to obtain the limit of radial load that may be applied by using attached piezoceramic annular plate. This will provide an information on significant radial load. Second part is the study of circular plate vibration problem which aim to look at the feasibility of using attached piezoceramic annular plate in tuning the circular plate frequency. For both, buckling and vibration problems, a parametric study was also performed. Results on vibration problem showed that the radial load applied through attached piezoceramic annular plate gave a significant range of frequency tuning for most configuration reported. Parametric study for buckling problem showed that the inner radius has a significant influence in critical buckling voltage except for the case of annular plate is thicker than the circular plate. Also, it is found that for inner radius far enough from the outer radius, the circular thickness only influence the critical buckling voltage when the circular plate has thickness near the annular plate or smaller. Lastly, the critical buckling voltage increases as the annular thickness increases regardless the annular plate is thicker or thinner than the circular plate. On the other hand, parametric study for vibration problem showed that the fundamental frequency is independent of the radius if the annular plate has equal thickness as the circular plate. While the similar may be conclude for the case where the annular is thicker than the circular plate but only for the if the inner radius is less than half of the outer radius. The fundamental frequency reduces rapidly as the inner radius became larger than half of the outer radius. For the case where the annular plate is thinner than the circular plate, the fundamental frequency increases with the change of the inner radius.

خلاصة البحث

المعادلة المسيطره التي تحكم اللوحة الدائرية و الحلقية للمادة المتجانسة قد تم استحداثها. المعادلات المسيطره قابلة للتطبيق عند الالتواء و مشاكل الاهتزاز خارج الطائرة. و أجريت دراسة على اللوحة الدائرية مع التردد الانضباطي. ويتحقق ضبط تردد من خلال ربط اللوحة الحلقيه بيزو السيراميك على حافة الصفيحة الدائرية. عن طريق التحكم في الجهد المطبق على اللوحة الحلقي بيزو السيراميك ، وبالتالي فإن تحميل شعاعي ، ويمكن ضبطها تواتر الأساسية من لوحة دائرية. يتم دراستها دراسة التواء من اللوحة الدائرية مع اللوحة الحلقية بيزو السيراميك و تعلق على حافته لإظهار جدوى هذه الفكرة. أظهرت النتائج من خلال التحكم في تطبيق الجهد ، وعموما الحمل التواء من اللوحة الدائرية تغيرت مما يدل على جدوى الأسلوب في التردد الأساسي من خلال ضبط تطبيق الحمولة الشعاعية عبر اللوحة الحلقي بيزو السيراميك. وهناك مشكلة من الاهتزاز من لوحة دائرية أن اللوحة الحلقي بيزو السيراميك تعلق على حافته تحت حمولة شعاعي و درس أيضا. ومن المعروف أن وتيرة الأساسية يذهب إلى الصفر في التواء الحمل. اللوحة دائرية حالة و حلقي سمك متساوون في 0.3 ملم يظهر تردد الأساسية لتكون قادرة على ضبط ما يصل إلى 0 هرتز. ومع ذلك ،لسمك مختلف، هو أنه أظهر أن تكوين لوحة الحلقي يلعب دورا هاما في فعالية تحميل شعاعي المطبقة في ضبط التردد الأساسي. أكبر عرض أو سمكا من لوحة الحلقي سيعطي التغيير أكثر أهمية في التردد الأساسي في تأثير الحمل شعاعي تطبيقها. ويرجع ذلك إلى الجهد القسرية من لوحة بيزو السيراميك القيد.

APPROVAL PAGE

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DECLARATION

I hereby declare that this dissertation is the result of my own investigations, except where otherwise stated. I also declare that it has not been previously or concurrently submitted as a whole for any other degrees at IIUM or other institutions.

Syed Noh Bin Syed Abu Bakar

Signature .... Date 9th July 2015

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…………………………….	9^{th} July 2015
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First and foremost, gratitude and appreciation is for Allah, the Most Merciful and Most Compassionate for granting me a precious opportunity to complete this work and granted me health and strength for the realization of this endeavor.

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LIST OF SYMBOLS

N_0	Applied Radial Load
V	Applied Voltage
$D^{(a)}$	Bending Stiffness of Annular Plate
$D^{(b)}$	Bending Stiffness of Bimorph Circular Plate
$D^{(s)}$	Bending Stiffness of Isotropic Solid Circular Plate
κ_{22}	Circumferential Curvature
θ	Circumferential Direction/Angle
N_{22}	Circumferential Load
e_{22}	Circumferential Strain
e_{22}^{0}	Circumferential Strain at Mid-surface
s_{ij}^{E}	Compliance Constants at Constant Electric Field
ρ	Density of Isotropic Plate
ρ_p	Density of Piezoceramic Plate
ϵ_{ij}^{T}	Dielectric Constants at Constant Stress
v_0	Displacement of Mid-surface in Circumferential Direction
u_0	Displacement of Mid-surface in Radial Direction
w_0	Displacement of Mid-surface in Thickness Direction
D_i	Electric Displacement Components
E_i	Electric Field Components
ϕ	Electric Potential
h_a	Half Thickness of Annular Plate

h	Half Thickness of Isotropic Layer/Plate
r_i	Inner Radius
r_o	Outer Radius
d_{ij}	Piezoelectric Constants
k_p	Planar Electromechanical Coupling Coefficient
ν_p	Poisson Ratio for Piezoceramic Plate
μ	Poisson ratio for the isotropic shim
κ_{11}	Radial Curvature
r	Radial Direction
N_{11}	Radial Load
e_{11}	Radial Strain
e_{11}^{0}	Radial Strain at Mid-surface
κ_{12}	Shear Curvature
N_{12}	Shear Load
e_{12}	Shear Strain
e_{12}^{0}	Shear Strain at Mid-surface
$e_{ij}^{(p)}$	Strains Components
$A^{(b)}$	Strecthing Stiffness of Bimorph Circular Plate
$\sigma_{ij}^{(p)}$	Stresses Components
$A^{(a)}$	Stretching Stiffness of Annular Plate
$A^{(s)}$	Stretching Stiffness of Isotropic Solid Circular Plate
z	Thickness Direction

h_p	Thickness of Piezoceramic Layer
q	Total Charge
E	Young coefficient for the isotropic shim

Subscript

P	piezoelectric

Supercript

(a)	Annular pieoceramic plate
(s)	Solid circular isotropic plate

CHAPTER ONE

INTRODUCTION

1.1 OVERVIEW

In this chapter, problem statement, background and motivation for the current research are presented. The chapter started with presentation of the motivation and background of the research. From the background, a problem statement is raised and a philosophy to the research solutions is presented. Then, the scope of the research is briefly explained. In order to answer the problem statement, several objectives are laid down. The approach of current study is explained in research methodology. The chapter concluded with thesis outline and the contributions of current study.

1.2 INTRODUCTION

Vehicles must consume fuel to supply the energy needed to move the vehicles and their passengers. The greater force needed to move the vehicles, the greater fuel will be consumed. One of the forces that contributed to greater fuel consumption is the aerodynamic drag. Aerodynamic drag exerts a force on the vehicles in the opposite direction from the velocity. It is known to be a principal determinant of energy consumption in the vehicles such as aircrafts because they operate at such high speeds. One flow phenomenon that contribute to increase of aerodynamic drag is known as flow separation. It is a phenomenon where the fluid flow becomes detached from the surface of the object or surface, and instead takes the forms of eddies and vortices. It is of interest many researchers to eliminate the separation or at least to delay the separation in order to reduce drag, hence reducing the fuel consumptions.

Method of separation control may be classified into passive and active approach. Passive approach is achieved by design an optimal aerodynamic shape of the vehicle body or by attaching structure such as wingtip and shark fin that may delay the flow separation. On the other side, active approach is achieved by attaching structure that may interact with the flow depending on the flow conditions. In passive approach, the structure is designed to handle flow separation for certain range of velocity. Outside this range the approach may be ineffective or even may be negatively influence the flow past the objects (i.e. create larger drag). On the other hand, active approach is more beneficial since it interacts with the flow depending on the flow conditions. Even better, with the advancement of current sensor-actuator technology and computer technology, the system may be designed to adapt to the surrounding automatically without the interference of human.

One of the examples of the concepts in active flow control is steady injecting momentum and removing mass from the boundary layer (near surface flow field). Typical example of researches in steady injection of momentum is by continuous blowing, known as surface-tangential blowing. While it may found way to production aircraft but it plagued by technical complexity and additional weight resulting from plumbing systems. Meanwhile a typical example of removing mass is by continuous suction. Similar to blowing, suction also has complex piping systems and also add weight. However, in recent years, there is an emergence of new technology that combined the concepts of injecting and removing which is known as synthetic jet actuator (SJA). An SJA also known as a zero-net-mass but non-zero momentum fluid flux generated by an oscillating structure such as a piezo-oscillator.

Research work on SJAs has shown great potential of using SJA in active control of boundary layer separation in order to reduce the drag and increase the efficiency of aerodynamic devices. The SJA consists of an oscillated boundary (i.e. electrically oscillated membrane and moving piston) located at the bottom of a small cavity which has an orifice in the face opposite the membrane.

Many research works have shown that the actuation frequency of the SJA plays a significant role in optimum separation control. Since for moving vehicles, the speed of the vehicles is changing throughout the travelling course, the flow regime or pattern also changing. Therefore, in order to have an optimum design of SJA, there is need to design an SJA that its actuation frequency may be tuned depending on the vehicle speed.

For an SJA that its oscillating boundary is a piezoceramic plate, the plate frequency may be tuned by several ways. Some of the researchers have showed that the natural frequencies may be control via shunt circuit (Davis & Lesieutre, 2000; Hagood & Flotov, 1991). However, there is limited range of frequency that may be tuned depending on the shunt circuit properties such as the capacitance. On the other hands there are also some researchers that showed how the in-plane load influenced the natural frequencies (Hebert & Lesieutre, 1998; Hu, et al., 2007; Lesieutre & Davis, 1997). Their study was limited to either on beam structure or by using complicated mechanical structure to provide the in-plane load.

1.3 PROBLEM STATEMENT AND ITS SIGNIFICANTS

Given the background of the study from the previous introduction, one would question whether the range of the tunable frequency may be broadened by using applied in-plane load without having complicated mechanical structure. To answer this question, one aims to study the idea of applying in-plane loads via electrical means in controlling the natural frequencies, therefore eliminating complicated mechanical structure. By eliminating the needs of mechanical structure, one would expect reduces in weight and/or size of a structure. For industry that put significant effort in reducing weight and size such as aerospace industry, the benefit of reducing weight and size would present a significant contribution.

To achieve the aim, one would study the effects of the in-plane load to natural frequency of a circular plate. The in-plane load is provided by using piezoceramic material as part of the circular plate structure. The piezoceramic material is used as a medium to provide the in-plane load through applied voltage.

1.4 RESEARCH PHILOSOPHY

SJA has emerged as an interesting method in controlling separation. However, it still did not found its way to actual application in aircraft industry or in any aerodynamics related devices. One of the issue related in designing an actuator as SJA is the effectiveness of its application in wide range of flow conditions. One of the factors that influence the effectiveness SJA is the frequency of the vortex trains that are produced by SJA. The frequency of the vortex trains are influenced by the frequency of the vibrating boundary that produced the vortex trains. Therefore, broadening the operating frequency of the vibrating boundary will also improve the performance of the SJA.

For SJA that the vibrating boundary is a piezoceramic plate, one may control the frequency via shunt circuit or apply in-plane load. While shunt circuit application limited to narrow frequency band depending on the passive design of the shunt circuit, the application of in-plane load riddle with complicated mechanical structure. It is desirable that the complicated mechanical structure may be removed while the frequency range may be broadened. The issue may be resolved if the in-plane load may be produced through electrical means. One way to produce the in-plane load is by using piezoceramic materials since the piezoceramic material has ability to convert electrical current to mechanical strains.

1.5 RESEARCH SCOPE

In current study, one would study vibration of circular plate structure under the influence of in-plane load. As part of the circular plate piezoceramic material is used as the medium of conversion of electrical voltage to mechanical strains (thus in-plane load).

This study is conducted through numerical method called Finite Difference Method. However, variables such as electric distributions and the in-plane loads are derived analytically. The derived variables are used with combination with the classic governing equations that govern a circular plate buckling and vibration problem to complete the numerical study. The obtained governing equations in FDM formulation is coded in MATLAB®. The validity of the MATLAB® code is validated by comparing the ability of the code to produced results for problems in available literatures.

1.6 RESEARCH OBJECTIVES

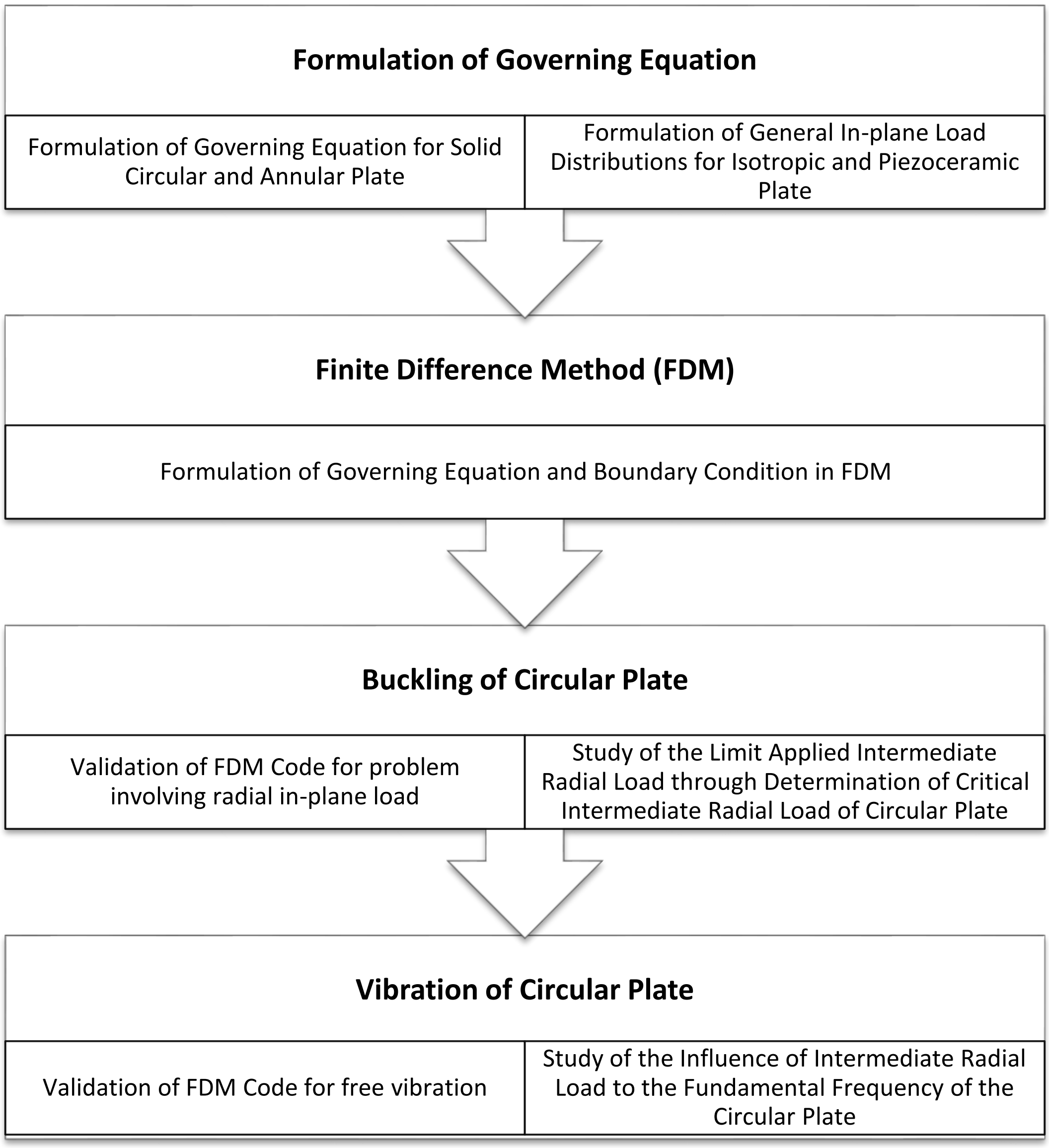
The specific objectives of this study are:

i. Formulate equations and load distributions that govern vibration problem of a circular plate with applied intermediate radial load using piezoceramic.
ii. Develop and validate finite difference code.
iii. Apply the finite difference code for buckling and vibration problem of circular plate with applied intermediate radial load.

iv. Determine the influence of intermediate in-plane load from electrical means to the natural frequency.

1.7 RESEARCH METHODOLOGY

The research methodology is summarized in the following chart.



In order to achieve the objective one would approaches the circular plate problem numerically by using finite difference method that is coded in MATLAB. In brief, one would first formulate the required load distributions and governing equations that governed a circular plate and an annular plate. Next, the formulated governing equations are converted into FDM formulations. Later the FDM formulation is coded in MATLAB and validated with the available case from literature. Once validated, the code is used to study the buckling and vibration of circular plate under application of intermediate radial load. First the buckling study is done to study the limitation of the plate structure. Then, the vibration study is performed to show the influence of the in-plane load to the natural frequency.

The formulation of the required expression and equations is explained in detail as in Chapter 3. Here is brief explanation on the formulation. First, fundamental assumptions are laid out. Secondly, the equations and expressions that involved in circular plate study are formulated. The formulation is divided into two parts which are formulation for solid circular plate and formulation for annular plate. In each parts, one starts with the definition of strain-mechanical displacement and constitutive equation. This follows with the derivation of general expression of force and moment resultants. The force resultants are used for deriving the expression of applied in-plane radial load. On the other hand, the definition of bending stiffness is obtained from moment resultants. The formulation continues with deriving expression of in-plane radial load for current study. Then, the governing equations for buckling and vibration are defined. Third part of the formulation is defining the boundary conditions that appear in this dissertation.

Next step is converting the formulations into finite difference expressions. In formulation of FDM, one starts with the treatment at the center of solid circular plate. Next is applying FDM to governing equations and boundary conditions. Lastly, the equations in FDM are transformed in matrix form. The theoretical of circular plate vibration and buckling is explained and the formulation is developed.

After formulating the expressions and equations in finite difference, the formulation is coded in MATLAB®. The code is validated by using existing problem in literature for both buckling and vibration of circular plate. Two problems are chosen which are an annular plate buckling due to the uniform compression at both its edges and buckling/wrinkling due to tensional load at its inner edge while its outer edge is free from loading. The main purpose is to show the FDM may handle the different type of in-plane load which one is constant throughout the plate while the other vary with radius. While the analysis is done for validating purpose, the two examples are also chosen to show the different of the stresses (thus the in-plane loads) distribution throughout the annular plates which is difference due to different nature of applied load and it may be dependent on the radius. These validations are done at the beginning of Chapter 4. On vibration part, another two examples chosen which are free vibration of clamped circular plate and influence of radial edge load to free vibration of clamped circular plate. Both examples should be suffice to show that the coded FDM is able to solve free vibration problem involving radial in-plane load. These validations are shown in early part of Chapter 5.

Once the FDM code is validated, it is ready to be used as tool to analyze circular plate problem. First, one would focus on study of circular plate buckling provide limitation of the size of the intermediate radial load that may be applied to the circular plate structure (last part of Chapter 4). The intermediate buckling load is realized by application of annular piezoceramic plate.

Lastly, problem of circular plate free vibration is studied (Chapter 5). First, the free vibration analysis is done without the effects of the in-plane load. Later, a circular plate vibration analysis also done to see the effects of the in-plane load on the fundamental frequency.

1.8 OVERVIEW OF THESIS

1.8.1 Thesis Outline

This thesis is organized as follows:

i. Chapter 2 provides the literature review on the related topics of current thesis.

ii. Chapter 3 provides the formulation that required in current study such as distribution of in-plane loads, governing equations and boundary conditions. This chapter also provided the FDM formulation. The finite difference method is applied to the formulated governing equations and boundary conditions. Issues involving FDM, such as treatment at the center of the plate, are also discussed.

iii. In Chapter 4, a problem of circular plate buckling due to the intermediate radial load is presented FDM code is validated. In early of this chapter, the FDM code is first validated. Two existing problems are solved by formulated FDM and the results are compared with the existing results for buckling. This also served as evident that the FDM code able to handle involving in-plane load.

iv. In Chapter 5, a problem of circular plate free vibration is presented. The validation of FDM code for free vibration of circular plate is also presented in this chapter.