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# ROBUST CONTROL DESIGN USING MODERN CONSTRAINED OPTIMIZATION TECHNIQUES

BY

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# ABSTRACT

Robust control design is commonly a difficult task that requires complicated mathematical formulation and heuristic parameters tuning. In addition, if often results in a high order controller. Motivated by the need to reduce complexity, a robust state feedback control design using modern constrained optimization algorithms is proposed in this thesis. Combining the advantages of robust control theory and computational intelligence makes the task more straightforward and automatic. Basically, a robust control design requires a set of goals to be achieved such as good transient response, zero steady state error for a constant input and most importantly, robustness to parameter uncertainty. A single-objective constrained optimization technique is used in the proposed method to handle these requirements. Searching for a set of robust controller gains that maximizes the stability radius of the closed-loop system is the objective. The constraint of the optimization is the region for the closed-loop poles that represents the desired time-domain control performance. In the beginning, the study is focused to find the suitable modern optimization tool(s) among the commonly used optimization tools such as Genetic Algorithm, Particle Swarm Optimization and Differential Evolution. The study further investigates the optimization features, such as constraint handling, stopping criterion and choice of optimization parameters. The result shows that Differential Evolution (with mutation factor=0.5 and crossover constant=0.9) outperforms Clerc’s Particle Swarm Optimization and Genetic Algorithm in constrained optimization problems. At the end of the study, the proposed robust control design using Particle Swarm Optimization and Differential Evolution are applied to pendulum-like systems, such as gantry crane, flexible joint and inverted pendulum. A set of laboratory experiments are carried out to evaluate the performance of the designed controller. LQR-based controller and $H_\infty$ loop-shaping controller are also designed for comparison with the proposed controller. The advantage of the proposed controller design is the automated tuning process for the controller parameters as compared to the benchmark controllers. Another contribution of the thesis is the dynamics modeling of the pendulum-like systems where a generic model structure for the pendulum-like systems is developed. The generic model structure is obtained by linearization around equilibrium and simplification where the dynamics effect of vibration to actuator dynamics is neglected. As a result, the parameters of the pendulum-like systems model can be easily identified by decoupling of vibration model and actuator model in the experiment.

# خلاصة البحث

تصميم السيطرة المتينة هو عمل صعب لأنه يطلب إفراط أعمل الرياضى و تقوم توليف البارامیتر تجربيا. يُدفع بالحاجة ليُخفَّف التعقيد يقترح في هذا البحث تصميم التحكم الغذية المرتدّة المتينة بإستخدام الغورتم الحديثة لتحقيق الوضع الأمثل. الأجتماع بين فضائل النظرية السيطرة المتينة و الأستخبارات الإحتسابى يبسّط الأعمال. في جوهر الأمر تصميم السيطرة المتينة يحتاج إلى اداء مجموعة من الأهداف مثل الإستجابة الإنتقالية الصالحة و صفر غلطة حالة مستقرة لإدخال ثابت و أهمّ هدف هو المتانة إلى إرتياب البارامتر. ليحقق هذا المنطلب يُستعمل في الطريقة المقترحة التقنيات لتحقيق الوضع الأمثل المقيدة بهدف واحد. فلهذا تحقيق الوضع الأمثل هدف و قيد. هدفه البحث عن مجموعة من نتائج جهاز التحكم المتينة التى تعلّى ذراع إستقرار البرنامج المختوم الذاتى و قيّده ناحية الأعمدة المختوم الذاتي التى تصوّر تحكم الأداء المطلوب. فى أول البحث يركّز الاهتمام فى إجادة أصلح الأدة الحديثة من الأدوات الحديثات التى يعود إستعمالها لتحقيق الوضع الأمثل مثل الغارتم الوراثي و تحقيق الوضع الأمثل 'سرب الجزئ' و التطور التفاضلى. فضلا عن ذلك يحقق البحث معالم تحقيق الوضع الأمثل و هي تصرف القيد و نظام الإيقاف و إختيار بارامتر تحقيق الوضع الأمثل. تعرض النتائج أن التطور التفاضلى (عامل التحويل= 0.5 و مقدار ثابت للتحويل = 0.9) هو أحسن من تحقيق الوضع الأمثل 'سرب الجزئ' لكلرك و التطور التفاضلى فى مشكلة تحقيق الوضع الأمثل المقيدة. في آخر البحث تصميم السيطرة المتينة المقترح بإستحدام تحقيق الوضع الأمثل 'سرب الجزئ' و التطور التفاضلى يُطبق في أنظمة تشبه الرقاص مثل مرفاع قنطرى متحرك و وصلة مرنة و رقاص منعكس. تقوم التجربات المعملية ليحلل أداء التحكم المتصم. فضيلة تصميم التحكم المقترح هو توليف البارامتر تلقائيا. و مساهمة هذا البحث الأخرى هي صيغة الأنظمة الديناميكية تشبه الرقاص أينما تطور تركيب الصيغة الشاملة لأنظمة تشبه الرقاص. تحصل تركيب الصيغة الشاملة بتخطيط التوازن و تبسيط أينما همل أثر ديناميكى عند ديناميات الأهتزاز إلى وحدة التشغيل . و النتيجة هي تحقق بارامتر الأنظمة القاص سهلا بطريق فك إقتران صيغة الأهتزاز و صيغة الوحدة التشغيل في التجربة.

# APPROVAL PAGE

The thesis of Mahmud Iwan Solihin has been approved by the following:

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# DECLARATION

I hereby declare that this dissertation is the result of my own investigations, except where otherwise stated. I also declare that it has not been previously or concurrently submitted as a whole for any other degrees at IIUM or other institutions.

Mahmud Iwan Solihin

Signature:……………………… Date: ……………………

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# LIST OF ABBREVIATIONS

| | |
|---|---|
| AGC | Anti-swing gantry crane |
| ALPSO | Augmented Lagrangian PSO |
| CI | Computational Intelligence |
| DC | Direct Current |
| DE | Differential Evolution |
| DEFC | DE-based State Feedback Controller |
| DOCHM | Dynamic Objective Constraint Handling Method |
| EA | Evolutionary Algorithm |
| FA | Firefly Algorithm |
| FE | Number of Function Evaluations |
| FJM | Flexible Joint Manipulator |
| GA | Genetic Algorithm |
| HS | Harmony Search |
| LMI | Linear Matrix Inequality |
| LQG | Linear Quadratic Gaussian |
| LQR | Linear Quadratic Regulator |
| LQRFC | LQR-based Feedback Controller |
| MIMO | Multi-input Multi-Output |
| PID | Proportional-Integral-Derivatives |
| PLS | Pendulum-like Systems |
| PO | Percentage of Overshoot |
| PSO | Particle Swarm Optimization |

| | |
|---|---|
| PSOFC | PSO-based State Feedback Controller |
| RIP | Rotary Inverted Pendulum |
| SA | Simulated Annealing |
| SISO | Single-input and Single-output |

# LIST OF SYMBOLS

| | |
|---|---|
| $C_r$ | crossover constant for DE |
| $\hat{E}$ | row scale matrix of perturbation structure for closed-loop system |
| $F_{crit}$ | critical value of $F_m$ for DE |
| $F_i$ | force input for gantry crane |
| $F_m$ | mutation factor for DE |
| $G_d(s)$ | disturbance transfer function |
| $G_p(s)$ | peturbed plant transfer function |
| $G_s(s)$ | shaped plant transfer function |
| $\widehat{H}$ | column scale matrix of perturbation structure for closed-loop system |
| $J_{arm}$ | moment of inertia of the arm for flexible joint |
| $J_{eq}$ | equivalent moment of inertia at the motor output for flexible joint |
| $K_e$ | electric constant for DC motor |
| $K_{stiff}$ | stiffness constant for flexible joint |
| $K_t$ | torque constant for DC motor |
| $L_a$ | inductance of armature for DC motor |
| $N_P$ | number of population (population size) |
| $N_{bit}$ | number of bits for GA |
| $P_c$ | crossover rate for GA |
| $P_m$ | mutation rate for GA |
| $R_a$ | resistance of armature for DC motor |
| $T_m$ | motor output torque |

$V_{max}$ maximum velocity of particle for PSO

$W_1(s)$ weighting function for shaped plant

$W_2(s)$ weighting function for shaped plant

$a_0$ numerator constant for $G_1$

$a_2$ numerator cosntant for $G_2$

$b_0$ denominator constant for $G_2$

$b_1$ denominator constant for $G_1$

$b_2$ denominator constant for $G_2$

$b_{eq}$ equivalent friction coefficient

$c_1$ constant for ”cognition’ part of PSO

$c_2$ constant for ”social” part of PSO

$f_1$ benchmark function 1

$f_2$ benchmark function 2

$f_3$ benchmark function 3

$i_a$ armature current of DC motor

$j_{max}$ maximum number of iteration

$l_b$ lower bound of solutions

$m_1$ payload mass for gantry crane

$m_2$ cart mass for gantry crane

$r_a$ length of the arm for flexible joint

$r_c$ complex stability radius

$r_p$ combined radius of gear and pulley for gantry crane

$u_b$ upper bound of solutions

$\sigma_d$ standard deviation

$b$ friction coefficient

| | |
|---|---|
| $\mathbb{C}$ | set of complex numbers |
| $\mathcal{F}$ | fractional transformation |
| $h$ | horizontal position of trolley/cart of gantry crane |
| $\mathcal{L}$ | Lagrangian equation |
| $\mathbb{R}$ | set of real numbers |
| $D$ | dimension of optimization problem |
| $E$ | row scale matrix of perturbation structure |
| $F$ | auxiliary function |
| $G(s)$ | nominal plant transfer function |
| $H$ | column scale matrix of perturbation structure |
| $J$ | performance index function |
| $K$ | controller gain vector |
| $L$ | length of pendulum |
| $Q$ | state weighting matrix of LQR |
| $R$ | input weighting matrix of LQR |
| $S(s)$ | sensitivity function |
| $T(s)$ | complimentary sensitivity function |
| $X$ | solution vector in optimization |
| $b$ | friction coefficient |
| $f$ | fitness/objective function |
| $g$ | gravitational acceleration |
| $j$ | index for iterations |
| $l$ | length of payload cable for gantry crane |
| $m$ | mass of pendulum |
| $q$ | generalized notation for linear position or angular position |

$v$ voltage input for motor

$w$ inertia weight of PSO

$x$ state variable vector for state space representation

$\mathcal{T}$ kinetic energy

$\mathcal{V}$ potential energy

$\mathbb{F}$ feasible region in optimization

$\mathbb{S}$ search space in optimization

$\Delta$ perturbation model to linear system

$\alpha$ angular position of rotating arm (inverted pendulum and fexible joint)

$\gamma$ tip angular position of flexible joint

$\varepsilon$ standard deviation threshold for stopping criterion in optimization

$\zeta$ damping ratio

$\eta$ number of iteration for which stopping criterion applies in optimization

$\theta$ swing angle (deflection angle)

$\lambda$ eigenvalue

$\mu$ structured singular value

$\rho$ ”transient margin”

$\sigma$ singular value

$\varphi$ angle in closed-loop poles region where $\varphi = \cos^{-1}(\zeta)$

$\psi$ wedge region for closed-loop poles

$\epsilon$ stability margin

$\vartheta$ robust performance level

# CHAPTER ONE

# INTRODUCTION

## 1.1 OVERVIEW

In classical control design techniques, difficulties arise when the plant dynamics are complex and poorly modeled, or when the performance specifications are particularly stringent (Green and Limebeer, 1995). Even if an optimal solution is eventually found, the process is likely to be time-consuming.

Hence, model error has been an important issue in linear control theory. Robust control theory has been used to deal with this particular issue besides robust stability and robust performance issues. A successfully designed controller should be also able to maintain stability and performance level in spite of uncertainties in system dynamics and/or disturbances to a certain degree.

Various robust controller design techniques have been proposed such as $H_\infty$, $\mu$-synthesis, etc. The robustness issue was just prominently considered in early 1980s with the pioneering work on robust control theory (Zames, 1981; Zames and Francis, 1983). This robust control is now popularly known as $H_\infty$ robust control. This approach to date is commonly used with techniques such as $H_\infty$ loop shaping (McFarlane and Glover, 1990) and $\mu$-synthesis/analysis. However, the theory behind the approach is not trivial. It is not straightforward to formulate a practical design problem into $H_\infty$ or $\mu$ design framework. In addition, standard robust control design can result in high order and complicated controller structure, which is difficult to implement in practice (Lin et al., 2009).