INTERACTIVE LEARNING – MATHEMATICA ENHANCED VECTOR CALCULUS (ILMEV)

BY

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INTERNATIONAL ISLAMIC UNIVERSITY MALAYSIA

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ABSTRACT

This dissertation presents a computer learning assistant ILMEV (Interactive Learning -Mathematica Enhanced Vector calculus), that helps students understand the theory and applications of integration in vector calculus. Many students in engineering, the physical sciences and mathematics are required to learn vector calculus and find the subject challenging. Learning assistants in such advanced applied mathematics are rare and present many challenges. First, the presentation of this material in textbooks is not suitable for an automated assistant and thus, had to be substantially reorganized. Additionally, no computer algebra system has algorithms powerful enough to automatically solve all but the most elementary problems of this type that appear in textbooks. To overcome this, we implemented a geometric modeler and a simplified version of the cylindrical algebraic decomposition (CAD) algorithm so that ILMEV can now compute closed form solutions to many two dimensional textbook examples as well as examples far more complex. ILMEV succeeds because it contains algorithms for reducing the integrals appearing in vector calculus to sums of iterated integrals and all computer algebra systems have powerful algorithms for computing iterated integrals in closed form. Vector calculus is complicated, requiring the correct application of many formulas which, in turn, requires extensive calculations involving calculus and algebra. It is vital that students acquire good intuition about complex geometric objects. Consequently, the design of an assistant must be based on the best pedagogical principles (interactivity, visualization and experimentation) to assist in the teaching and learning. One important aspect of ILMEV is that it proceeds in simple logical steps and explains what it is doing. All of this would be for naught if ILMEV does not have an easy to use interface that enables student access to the power of a computer algebra system.

ملخص البحث

هذا البحث يقدم لنا مساعدة الحاسوب في الأداء التعليمي Interactive Learning-Mathematica Enhanced Vector calculus (ILMEV). هذه الوسيلة تساعد الطلاب على فهم نظريات وتطبيقات التكامل في مادة الـ (vector calculus). هنالك تحديات تواجه استعمال الحاسوب في مادة الـ (vector calculus). إحدى هذه التحديات هي طريقة عرض الكتب لهذه المادة، الأمر الذي يحتاج إلى تفسير جذري. ثانياً: لا يوجد برنامج له المقدرة على حل جميع مسائل الجبره، وإنما كل البرامج الموجودة تساعد على حل جزئية فقط من مسائل الجبره. لحل هذه الإشكاليات، اقترحنا برنامج يسمى بـ ((Cylindrical Algebraic Decompositon (CAD)). هذا المقترح له المقدرة على وجود حل (closed form) لمعظم المسائل الموجودة في الكتب. برامج ILMEV نحج في تحويل التكامل إلى صيغة جمع التكامل المتكرر، والبرامج الحالية لها القدرة على حل مسائل التكامل المتكرر. ومن أهم مزايا هذه المقترح أنه يوضح كل خطوات الحل، كما أن هذا البرنامج سهل الاستعمال من قبل الطلاب.

APPROVAL PAGE

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DECLARATION

I hereby declare that this thesis is the result of my own investigations, except where otherwise stated. I also declare that it has not been previously or concurrently submitted as a whole for any other degrees at IIUM or other institutions.

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Signature .... Date ……………………….

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> Allah bears witness that *La ilaha illa Huwa* (none has the right to be worshipped but He), and the angels, and **those with knowledge** (also give this witness); He always maintains His creation in Justice. *La ilaha illa Huwa* (none has the right to be worshipped but He), the All-Mighty, the All-Wise (Qur'an, *Ali 'Imran*: 18).

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

ILMEV is a tool developed to assist students in learning how to perform integration over nontrivial domains in vector calculus. Vector calculus was chosen because it is critical for solving problems in engineering and science (many educational programs require a course on this topic) and the mathematics involved is difficult, even for talented and well-trained students. Many problems in three dimensional vector calculus involve the computation of integrals of functions involving vector fields on curves, surfaces and volumes. Such integrals can be reduced to integrals of scalar functions over an interval, area, or volume. However, these reductions require substantial manipulations involving calculus and algebra. Problems in two dimensions are technically easier to solve, but are, in fact, conceptually more difficult because both curve and surface integrals in three dimensions reduce to curve integrals in two dimensions. The reduction of several types of integrals are implemented in ILMEV.

Once the reduction to scalar integrals is completed, then these integrals need to be evaluated. Computer algebra systems contain powerful algorithms for computing one dimensional definite integrals analytically. These algorithms typically apply a large number of heuristics to a problem to try to obtain an answer quickly. If this fails then they apply the Risch algorithm (Geddes, Czapor & Labahn, 1992). If this algorithm cannot compute the integral, then an exact integral likely doesn't exist and the integral can only be evaluated numerically. By successive application, these algorithms can also compute multidimensional iterated integrals. Unfortunately, the multidimensional integrals that appear in vector calculus are typically not iterated except for problems that are geometrically very simple.

An important observation in this thesis is that, using a deep algorithm from computational algebraic geometry called cylindrical algebraic decomposition (CAD), it is possible to reduce integrals over regions in two and three dimensions to a finite sum of iterated integrals, allowing the solution of a wide range of problems. Current CAD algorithms are implemented as part of algorithms to perform quantifier elimination (QE), a feature that is not needed for integration algorithms. These black-box implementations are not consistent with educational needs, so in ILMEV we have reimplemented these algorithms for regions in the plane described by linear and quadratic equations. Such regions have already seen important applications in QE (Weispfenning, 2001). Extension to more complex regions and three dimensional problems is an ongoing project.

A powerful feature of vector calculus is that it possible to change certain special but important integrals over volumes to integrals over the bounding surface of the region and vice versa using the divergence theorem. It is also possible to change integrals over surfaces to integrals over the bounding curve of the surface and vice versa using Stokes' theorem. Although not typically included in this list, it also possible to reduce some curves integrals to evaluation of functions at the endpoints of the curve (sometimes called the potential theorem). These results are vital for understanding the applications of vector calculus and provide for easy evaluation of some complex integrals. These theorems all reduce to the fundamental theorem of calculus in one dimension. In two dimensions, the divergence and Stokes' theorems reduce to Green's theorem. These theorems will be implemented in ILMEV as an ongoing project.

Unfortunately, the results about vector calculus are not presented in textbooks in a form suitable for building a tool like ILMEV, so substantial effort and several iterations were required to accomplish this. Because of the complexity of the subject matter, the development of ILMEV demanded the careful application of educational principles (interactivity, visualization, experimentation and multiple representations) in the teaching and learning and an easy to use interface which adds features specific to its intended educational use.

1.2 AN OVERVIEW OF COMPUTER ALGEBRA SYSTEMS (CASs)

One of the important developments in computational techniques in the last forty years or so has been the evolution of computer algebra systems (CASs) or symbolic mathematical computer programs. These systems allow the direct manipulation of mathematical symbols and permit the possibility of exact, error-free solutions of long, complicated problems. Most systems also now include numerical and graphical features, which along with their symbolic capabilities and typically a user programming language, provide integrated computational environments (Wester, n.d.).

General purpose CASs, which are designed to solve a wide variety of problems, have gained special prominence, the most widely available of which are Derive, Maple, Mathematica and the Texas Instruments (TI) symbolic calculators such as the TI-89, TI-92 and Voyage 200. There are also a number of mathematical, statistical, educational and engineering packages whose main emphasis is typically not computer algebra, but which contain a symbolic engine within them such as Mathcad, MathView, Matlab, Scientific Notebook/Workplace and Symbolic SPICE. CASs and associated software are available in a variety of forms: commercially, as programs free for academic use, as shareware, under GNU Copyleft, and as public domain software (Wester, n.d.).

Akritas (1989) defined the term computer algebra (or symbolic and algebraic computation) as the capability of computers to manipulate mathematical expressions in a symbolic rather than numerical way, much as one does algebra with pencil and paper. By dealing mainly with exact numbers (infinite precision integers and rational numbers) and algebraic expressions in terms of their symbolic representations, CASs can free scientists from the painstaking concern for numerical errors (truncation and round-off) and thereby help to gain more insight into the various physical phenomena under examination. The purpose of computing is insight, not numbers. Insight is sometimes obtained by evaluating a mathematical expression, but in many cases the relations of the quantities are made clearer by algebraic means (Akritas, 1989).

Computer algebra has been applied to a variety of problems in numerous fields such as engineering, medicine, bio-technology, physics, chemistry, robotics and education (http://math.unm.edu/~aca). As in other active branches of science, the sociology of computer algebra is shaped by its leading conferences, journals and the researchers running them. The premier research conferences on computer algebra are the annual International Symposium on Symbolic and Algebraic Computation (ISSAC) and the International Conference on Applications of Computer Algebra (ACA). Additionally, there are also highly successful journals such as the Journal of Symbolic Computation (JSC), SIGSAM bulletin (ACM) and the International Journal of Computer Algebra in Mathematics Education. Some high-quality journals with a different focus sometimes contain articles in the field such as the Journal of Theoretical Computer Science, Journal of Mathematics of Computation, Journal of

Computer Physics Communications and many more (Joachim von zur Gathen & Gerhard, 1999).

CASs has become an important computational tool in the last decade. However, CASs was created by human beings and human beings are a common source of mistakes and inadequacies. In some circumstances, CASs are capable of giving inappropriate answers, incomplete answers, misunderstood answers and even wrong answers. On the other hand, most CASs are capable of giving an answer without any explanation on how the answer was obtained. The user has to determine whether the answer is correct or otherwise by using other means such as by stepping through the problem using pencil and paper. CASs are under continuous improvement, so some potential for error will always be expected (Aslaksen, 1999; Bernardin, 1999; Fateman, 2006a, 2006b; Gruntz, 1999; Stoudt, n.d.; Wester, 1999; Yuzita, 2000b).

In this research, we developed a multimedia symbolic package called Interactive Learning – Mathematica Enhanced Vector calculus (ILMEV) with the purpose of enhancing the process of problem solving in the area of advanced under-graduate calculus and also for a wider range of analysis problems. This package extends the capabilities of a CAS (Mathematica) in geometries that involves the computation of multidimensional integrals analytically and also assists users in the development of mathematical concepts and skills, mathematical problem solving processes and mathematical reasoning (Hamzah, Yuzita & Ahmad Faris, 2004).

1.3 RESEARCH BACKGROUND

The background of this research was based on the research done on the vector analysis packages that were embedded in six well known CASs which are Derive 6, Maple 9, Mathcad 11, Mathematica 5.2, MuPAD Pro 3.1.1 and REDUCE 3.8, and some not embedded in CASs such as Vector Calculus & Mathematica (Davis, Porta & Uhl, 1999), General Vector Analysis (GVA) (Qin, 1999) and vec_calc (Yasskin & Belmonte, 2003).

1.4 INTERACTIVE MULTIMEDIA IN THE TEACHING AND LEARNING OF MATHEMATICS

The use of computers to enhance the teaching and learning of mathematics is not a new concept. Since 1970, there had been a desire for computers to play more of a role in education (Waddick, 1994). For instance, computer-assisted learning has shown a significant and positive impact towards the user's attitude and syllabus content (Roblyer, 1998). Additionally, multimedia technology using computers provides an enjoyable and interesting learning environment for the users. It has changed the traditional learning environment to a technological learning environment. On the other hand, interactive learning is also not a new pedagogic approach. About twenty five hundred years ago, Socrates used this approach during question and answer sessions to promote active thinking among his students. Nowadays, interactive multimedia is widely used using Socrates' approach (Murphy, 2002).

In this research, two main interactive characteristics suggested by the Center for Excellence in Education (CEE) Publication (1998) were incorporated in the implementation of ILMEV:

i. The methods and results are represented in the form of non-linear presentation. The user has control over when to execute a particular method or step that is included in a succession of steps by clicking on a particular button such as compute button, in order to perform a task. All the steps have been programmed earlier.

ii. The user has control over a presentation by making an appropriate choice such as choosing a button (e.g., $\iint_{\Omega}$ dA button) in the main menu which consists of a few other buttons (e.g., $\int_c$ ds, $\int_c$ **F•t** ds, $\int_c$ **F•n** ds buttons). However, the program has level of protection and still has to maintain control over some parts of the program such as the decision to what can and cannot be input by the user.

Additionally, the Learning Without Frontiers (LWF) Homepage (2002) defined interactivity as a feedback process for the exchange of information between two or more parties (e.g., a user and ILMEV) in the communication or learning environment. On the other hand, Maras (n.d.) defined interactivity as the way multimedia technology represents its information in a particular user environment. This environment is a “trial and error” exploration field that has user-friendly navigation. In our case, we developed the ILMEV environment with this concept in mind.

Furthermore, Clark and Graig (Murphy, 2002) defined interactivity as the ability to give a correct feedback that will benefit the user. A system needs a good design to enable it to give a good response to user needs. Additionally, a system must also have the capability to figure out the user action and be able to translate and react or adapt to it. Duchastel (Murphy, 2002) added that the translation process is difficult for any system to do because it is such a complex process. In this case, ILMEV uses all the important capabilities that exist in the CAS (Mathematica) to provide the user with a proper feedback (such as the appropriate vector integration formula) and in places, some extensions were done to perform a particular computational task.

Additionally, Interactive Learning System (ILS) is also a term used to cover a wide range of learning situations in which various types of knowledge or information exchange take place between communicating partners that are involved in some form of dialogue process. Systems that integrate the use of many different instructional technologies are often referred to as multimedia learning systems. An ILS consists of three basic components: a learner population, a delivery system and the pedagogic material that is to form the basis of learning. Systems that facilitate interactive learning must be responsive, adaptive and dynamic with respect to both the needs of the learner population and the mechanisms of knowledge transfer that they employ (Barker, 1990).

Multimedia is defined as a combination of texts, graphics, sound, animation, still images and video that is sent by a computer or electronic device (CEE publication, 1998; Educational Resources Multimedia '94 Catalog, 1994). Additionally, NCSU (1999) defined multimedia as a computer interaction with human beings that involves texts, graphics, voice and video. Usually, hypertext concepts are also instilled in it. Therefore, we can refer to interactive multimedia as multimedia that allows control by its users. In other words, interactive multimedia defines both the content and context of information, education and entertainment that can be accessed, manipulated and translated by users. The interactive multimedia environment is a dynamic environment. The environment and process change based on situation, learning context and individual needs (Giardina, 1992). Furthermore, Wilson (1992) referred to interactive multimedia as a tool and activities which enable presentation of information to be integrated electronically and allow user control over various types of information using formatted media such as video, still image, texts, graphics, animation, sound, numbers and data. In this research, we hope that the users' experience in using interactive multimedia will produce results from various perspectives by self-learning through activities such as exploration, manipulation and experimentation with the elements in ILMEV.

1.5 RESEARCH PROBLEMS

In this research, we focus our attention to these two following areas:

1.5.1 Two Dimensional (2D) Geometry: Computation of Multidimensional Integrals Analytically

There are some important computations and visualizations from analysis that are difficult to perform in all CASs such as computation of integrals over curves, surfaces and volumes and applying the multidimensional integration by parts theorems, usually called the potential, Green's, Stokes' and divergence theorems. The main difficulty in finding length, areas and volumes along with related physical quantities such as work, mass, flux and center of mass is geometric. The description and understanding of the geometry is a major stumbling block, although there are also many difficult algebraic problems even when the geometry is understood. CASs should be able to work with geometric regions in two and three dimensions that are described by inequalities. For example, let $F_i\ (x, y, z)$, $1 < i < k$ be smooth functions, where k is a positive integer and then define the sets

$$S_i = \{\ (x, y, z) \mid F_i\ (x, y, z) \geq 0\ \}$$

The sets S_i are closed rather than open because we have used $\geq$ rather than $>$. However, both open and closed sets should be understood by the CAS.

The intersection and union of two such sets are described as follows:

$$S_i \cup S_j = \{\ (x, y, z) \mid F_i\ (x, y, z) \geq 0\ \wedge\ F_j\ (x, y, z) \geq 0\ \}\ ,$$

$$S_i \cap S_j = \{\ (x, y, z) \mid F_i\ (x, y, z) > 0\ \vee\ F_j\ (x, y, z) \geq 0\ \}\ ,$$

where $\wedge$ means "and" and $\vee$ means "or" (Steinberg, 1999). A good CAS must be able to handle arbitrary intersections and unions of sets. In this research, we will just look at the closed intersections of n sets, where n is a positive integer.

When the functions F_i are polynomials, then the sets described above are called semi-algebraic. Such sets have been studied extensively, both theoretically and computationally. Hoon Hong, Richard Liska and Stanly Steinberg have written a survey (Hong, Liska & Steinberg, 1997a) and a technical article (Hong, Liska & Steinberg, 1997b) describing applications to stability theory involving such sets. These applications use an algorithm called Quantifier Elimination by Partial Cylindrical Algebraic Decomposition (QEPCAD) (Collins & Hong, 1991) which takes statements made up from polynomial equalities and inequalities, logical connectives, and quantifiers, and produces a logically equivalent formula free of quantifiers and the quantified variables. One conclusion of the study is that cylindrical decomposition plays a central role in our geometric problems and that algorithms for doing cylindrical decompositions are needed for at least the class of sets described by elementary transcendental functions (Steinberg, 1999). In this research, we are not interested in quantifier elimination directly, but in the cylindrical algebraic decomposition (CAD) part of the QEPCAD algorithm.

For solving our problems, we need a good geometric modeling program. There are some commercial CASs such as Mathematica, the TI-92 and Voyage 2000, that are able to do some geometric modeling but they are only adequate for simple examples. These CASs understand intersections and unions of sets defined by polynomial inequalities of the form $P(x, y) > 0$ and $P(x, y) \geq 0$. However, no commercially available CAS contains algorithms for doing cylindrical decomposition other than QEPCAD (Hong, Liska & Steinberg, 1997a; Steinberg, 1999) and