



FINITE DIFFERENCE TIME DOMAIN METHOD TO INVESTIGATE ELECTROMAGNETIC FIELD EFFECTS IN HUMAN BODY

BY

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A dissertation submitted in partial fulfilment of the requirements for the degree of Master of Science in Computer and Information Engineering

Kulliyyah of Engineering International Islamic University Malaysia

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ABSTRACT

Maxwell's equations are implemented using Finite Difference Time Domain (FDTD) method to investigate the radiation effects in human body. Flat Phantom Model for human body is proposed using incident electric fields and Perfect Matched Layers (PML) boundary conditions. The proposed model has been implemented in Matlab Codes. The electric field distribution and Specific Absorption Rate (SAR) are calculated for all possible human organs using the measured electric field intensity at 900MHz, 1800MHz and 2.4GHz at International Islamic University Malaysia (IIUM), Gombak Campus. The values predicted by proposed method are found close to those calculated by the commercial Remcom Inc Software, XFDTD6.4. The effect of SAR has been predicted for different organs using proposed model. It is observed that the effect is higher in higher frequencies and the organ affects worse is brain. The SAR predicted by the proposed model for measured radiated fields at aforementioned frequencies are compared with safety guidelines given by the recognized body such as ANSI/IEEE, ICNIRP and Malaysia Communication and Multimedia Commission (MCMC). The predicted SAR is found 0.083W/Kg at 900MHz, 0.751W/Kg at 1800MHz and 1.434W/Kg at 2.4GHz which are 4.2%, 37.6% and 71.7% respectively of safety limits proposed by ICNIRP. The preliminary results show that the campus is safer for its inhabitants.

(SAR)

.(PML)

(SAR)

•

900

.(Matlab)

2.4 1800

•

•

(SAR)

.XFDTD 6.4 (Remcom)

.(MCMC) ICNIRP ANSI/IEEE 0.751 900 W/Kg 0.085 %4.2 2.4 W/Kg 1.434 1800 W/Kg

.

.ICNIRP %71.7 %37.6

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APPROVAL PAGE

I certify that I have supervised and read this study and that in my opinion, it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Master of Science in Computer and Information Engineering.

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DECLARATION

I hereby declare that this dissertation is the result of my own investigations, except where otherwise stated. I also declare that it has not been previously or concurrently submitted as a whole for any other degrees at IIUM or other institutions.

Hikma Shabani

Signature.....

Date

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This dissertation is dedicated to my parents and Brother Shahran Kassim who have been supported me all the way since the beginning of my studies. Also this thesis is dedicated to my wife and my children who have been a great source of motivation and inspiration. Finally, this thesis is dedicated to all who believe in the richness of learnig.

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TABLE OF CONTENTS

Abstract	ii
Abstract in Arabic	iii
Approval Page	iv
Declaration Page	v
Copyright Page	vi
Dedication	vii
Acknowledgements	viii
List of Tables	xi
List of Figures	xii
List of Abbreviations	xiv

CHAPTER 1: INTRODUCTION	
1.1 Background	1
1.2 Problem Statement	
1.3 Research objectives	4
1.4 Research methodology	5
1.5 Thesis outline	6

CHAPTER 2: ANALYSIS: ELECTROMAGNETIC FIELD RADIATION.... 8 2.1 Introduction

4	2.1	Introduction	8
2	2.2	Electromagnetic fields theory	9
		2.2.1 Maxwell's theory	10
		2.2.2 Maxwell's field equations	11
2	2.3	Plane wave propagation	13
2	2.4	Radiation Exposure Levels	17
2	2.5	Maximum Permissible Exposure	19
2	2.6	The Biological Effects Of Electromagnetic Fields	25
2	2.7	Summary	27

CHAPTER 3: FDTD FLAT PHANTOM MODEL

8
8
9
9
5
7
8
0
0
1

3.4.3 Boundary Conditions	43
3.5 Computational Algorithm	45
3.6 Matlab Simulation	48
3.7 Summary	51

CHAPTER 4: SETUP AND RADIATION MEASUREMENT	. 52
4.1 Introduction	. 52
4.2 Experimental Setup	52
4.2.1 Measurement Area and Radiation Source	52
4.2.2 Equipment used	. 55
4.3 Radiation Measurement: Method and Procedure	. 56
4.3.1 Mahallah Ali's Basketball Court	. 58
4.3.2 Celcom Building	. 59
4.3.3 Rectory Building	. 60
4.3.4 IIUM's Library	61
4.4 Measured Data	63
4.5 Summary	66

CHAPTER 5: RESULT ANALYSIS	67
5.1 Introduction	67
5.2 Evaluation of Measured Electric Field	67
5.3 Electric Fields Distribution Inside Human Body	70
5.4 SAR Analysis	78
5.5 Summary.	85
-	

CHAPTER 6: CONCLUSIONS, SUGGESTIONS, FUTURE WORK	86
6.1 Conclusion	86
6.2 Suggestions For Future Work	87

BIBLIOGRAPHY	7	88
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APPENDIX A: ELECTROMAGNETIC SPECTRUM	94
APPENDIX B: IIUM-LIBRARY'S Wi-LAN ANTENNAS' PLAN	95
APPENDIX C: Data from RFEX Software	99
APPENDIX D: 3-D FDTD Codes + PML Absorbing Boundary Conditions	106

LIST OF TABLES

Table No.		Page No.
2.1	ICNIRP/IRPA basic restriction radiofrequency and microwave radiation exposure for workers	22
2.2	ICNIRP/IRPA basic restriction radiofrequency and microwave radiation exposure for general public	23
2.3	ANSI/IEEE basic restrictions SAR	24
2.4	ICNIRP basic restrictions SAR	24
4.1	Specification of Tri-axis Probe	56
4.2	Generated Raw Data from RFEX Software for GSM900: Ali Basketball Court	63
4.3	Generated Raw Data from RFEX Software for GSM1800: Celcom Building	64
4.4	Generated Raw Data from RFEX Software for WLAN 802.11b: Central Library	64
5.1	Percentage for the highest peak measured Values over limits for GSM900, 1800MHz, and Wi-LAN	68
5.2	Densities and electrical properties of the simulated tissues.	72
5.3	Average Dielectric Properties of the equivalent Human tissues	78
5.4	Percentage for the highest simulated SAR Values over limits	79
5.5	Percentage for simulated SAR Values from Matlab over limits	82
5.6	Percentage for the highest peak simulated SAR Values over limits	83

LIST OF FIGURES

Figure No.		<u>Page No.</u>
2.1	Normal incidence at a planar boundary between two lossy media	13
3.1	Position of the field components in a unit cell of the Yee's lattice	31
3.2	Typical relations between field components within a quarter of a unit cell	33
3.3	Position of the electric field and magnetic field vector components about a cubic unit cell of the Yee space lattice.	34
3.4	Division of the FDTD lattice into total and scattered fields religion	42
3.5	Flowchart of FDTD Implementation.	46
4.1	Layout of the IIUM campus and Base Station locations.	53
4.2	Mobile Base Stations at Mahallah's rooftop.	53
4.3	The Artificial Tree at Mahallah Ali.	54
4.4	TS-EMF Equipments	55
4.5	Measurement Area of BasketBall Court at Mahallah Ali	59
4.6	Measurement Area at Celcom Building	60
4.7	Measurement Area at Rectory Building	61
4.8	Measurement Area at Level 4 in Central Library	62
5.1-12	Total electric field received at 900MHz	73
5.13-24	Total electric field received at 900MHz	76
5.25	SAR distribution in the Brain	80
5.26	SAR distribution in the Skin	81
5.27	SAR distribution in the Eye	81

LIST OF ABBREVIATIONS

Tree Dimensional Finite Difference Time Domain
Absorbing Boundary Condition
American National Standard safety levels
Code Division Multiple Access
Comité Européen de Normalisation Electrotechnique
(European Committee for Electrotechnical Standardization)
Continuous Wave
Digital Enhanced Cordless Telecommunications
Electromagnetic Field
Electromagnetic Radiation
Federal Communications Commission
Finite Difference Time Domain
Global System for Mibile Communication
International Commission on Non-ionizing Radiation Protection
International Radiation Protection Association
Institute of Electrical and Electronics Engineers
International Islamic University Malaysia
Local Area Network
Matrix Laboratory
Malaysian Communications and Multimedia Commission
Meter, Kilogram and Second
Method of Moment
National Council on Radiation Protection & Measurement
World Health Organization
Normal Mode Helical Antenna
Perfectly Matched Layer
Radiation Boundary Condition
Radio Frequency
Radio Frequency Radiation
Specific Absorption Rate
Transverse Electromagnetic Wave
Transmitter Systems for Electromagnetic Field
Universal Mobile Telecommunications System
Wireless Local Area Network

CHAPTER 1

INTRODUCTION

1.1. BACKGROUND

The telecommunications industry has been in a constant state of growth. Since the introduction of mobile phones in the mid-1980s, there has been a significant increase in the number of mobile phone users and installations of base stations. The statistics from the Malaysian Communications and Multimedia Commission (MCMC) show that the mobile penetration rate is 39 persons per 100 populations which are equal to 9.7 million subscribers (Ng. Kwan-Hoong., 2003). Nowadays, everybody can use mobile phones as the cost goes lower and the service expands more interactive. Around 80% of world's population has mobile phone coverage as of 2006. This Figure is expected to increase to 90% by the year 2010 (Wikipedia, 2006).

In order to support the communication system, more and more mobile base stations are built. The locations vary from telecommunication tower to the antennas mounted at rooftop of buildings. Nine base stations operated by three mobile services provider in Malaysia as Celcom, Digi and Maxis are located in International Islamic University Malaysia (IIUM), Gombak campus. Few stations with multiple antennas transmit signals at 900MHz, 1800MHz, and 2.4GHZ bands. According to Md. Rafiqul Islam (2006), Radiation exposes to 15000 population of IIUM campus including students and staff. Recently, all student hostels including Library have been upgraded with Wireless LAN facilities that installed a large number of antennas. Most of the radiation is indoor environment where students and staff are exposed almost all the time. Thus, with this wide spread use of wireless devices there has been an increasing

public concern about possible health hazards resulting from exposure to electromagnetic (EM) wave (Guy, C.K., 1986). Hence, it has been known since the early days of radio that Radio Frequency (RF) energy can cause injuries by heating body tissue. In extreme cases, RF-induced heating can cause blindness, sterility and other serious health problems. These heat-related health hazards are called thermal effects, J. A. Elder (1994). The proposition that these thermal effects may produce harmful health consequences has produced a great deal of research.

A group of scientists that working for telecommunication companies have shown that there is no relation between mobile phone and cancer or other diseases. They undergone all the scientific methods and they proved that mobile phone is totally safe for user (Dornan A., 2002). The confusing among the people is just like what happened in 1960s when the public were suspicious and unduly worried about the Xray exposure from their color television. In the 1980s, electromagnetic fields from computer terminal have been said to cause miscarriages, birth defects and other health problems, (A. Kangarlu and L. Robitaille, 2000). In the 1990s, with the Internet revolution, people have begun to raise their concerns over the Internet addiction among it users. Though, all the claims have been overturned.

On the other hand, the other independent group, their research is funded from independent body, or government body, are concerned about human health. Even though there is no scientific evidence that has been revealed, they have managed to prove several possibilities of health effects on human being. For example in October 2004, scientists at Karolinska Institute in Stockholm found that long term users of mobile phones were four times as likely to develop growth of brains tumors on the side they held the phone and twice as likely as non-users to develop these noncancerous growth, (S. Lönn, 2004). In 2003, study made by Professor Leif Salford shows that 2 hours exposure to mobile phone radiation destroy cells in important part of memory, movement and learning in rat's brain. It could also possibly happen because of the conceivably premature onset of illnesses such as Alzheimer. According to the research, this happens due to the mobile radiation, which allows harmful proteins, and toxins go through the brain barriers in rats, (W. Thomas 2003).

1.2. PROBLEM STATEMENT

The development of cellular communications and, in particular, the widespread use of mobile phones during the last few years have motivated a great social and scientific concern about possible harmful effects of the electromagnetic radiation from these sources on the user's health (Mart'ýnez . M. B., 2004). Therefore, the most significant parameter when discussing the health risks at these frequencies is the Specific Absorption Rate (SAR). Hence, the exact analytical solutions to estimate SAR are obtained only for simple scatters like the sphere and the circular cylinder which may be solved using separation of variable while for complicated scatters like most body organs the numerical method is used for accurate results (Allen T., Morris E., Brodwin., 1995). Several numerical models such as cylindroid whole body human phantom have been developed for calculating whole body electromagnetic absorption but only little information is available on the accuracy of these numerical solutions for complex biological bodies (Kayanagi Y., 2001). Thus the approaches to this problem consist of solving Maxwell's Equations in either differential or integral form. These approaches fall into two categories: time domain and frequency domain. The most successful frequency domain method is the method of moments (MoM) which requires computer storage and computation time on the order of $(3N)^2$ and $(3N)^3$, respectively, where N is the number of cells that may be on the order of several

thousand to tens of thousands. Among the time domain methods is the finitedifference method.

In contrast to MoM, the finite-difference time-domain (FDTD) method has storage and time requirements propotional to a constant time N. Even though N is considerably greater for the FDTD method because of an overhead of free space cells surrounding the body, when bodies of 20,000 cells are contemplated, any method whose resource requirements increase linearly rather than geometrically presents an attractive alternative (Dennis M., Sullivan., David. T. B., 1987). Therefore, a simple approach involves solving Maxwell's curl equations using a finite-difference approximation to both space and a time derivative has been proposed to evaluate the specific absorption rate (SAR) in human body.

1.3. RESEARCH OBJECTIVES

The research is proposed in order to overcome the aforementioned shortcomings and problems. The prime and foremost goal of this research is to determine either people are safer or not from Radiation by measurements and computational analysis of SAR in phantom models of human body placed inside IIUM compound.

The objectives of this research are:

- i. To develop an analytical phantom model of human body
- ii. To measure radiation (from GSM and Wi-LAN) inside IIUM compound
- iii. To investigate SAR and compare with safety regulations.
- iv. To produce a conclusion either the EM field in IIUM compound is compliance or not with safety limit by the authorized bodies

1.4. RESEARCH METHODOLOGY

This research has been completed in four steps as follow:

a) Modeling.

The modeling is done using the following steps:

- i. Maxwell's equations which are a set of linear partial differential equations in space and time domain
- ii. Finite Difference Time Domain (FDTD) study that is a numerical method used for solving partial differential equations in both space and time domain
- iii. Boundary Condition is applied due to the impossibility of simulating an infinte space; computer cannot store an unlimited amount of data
- iv. Development of Matlab code to plot the developed analytical phantom model of human body
- v. Electric field variation and Specific Absorption Rate (SAR) estimation using both developed Matlab code for the proposed phantom model and XFDTD6.4 software from Remcom Inc
- vi. Comparison of calculated values from proposed phantom model plotted into Matlab code with XFDTD6.4 (Remcom software)
- b) Measurement.
- i. The strategic locations have been selected based on the level of density of the students and staff in that particular area.
- ii. The portable field measurement system has been used to measure the electric field and power density from W-LAN and mobile based stations.

- c) Specific Absorption Rate (SAR) Estimation.
- i. The measured field has been used as incident field for the developed phantom model.
- ii. In order to estimate the effects on different parts of the body, the following parameters has been estimated:
- Maximum Specific Absorption rate (SAR) or local peak SAR distributions.
- Whole body SAR Average.
- d) Comparison and Recommendations.
- The estimated SAR values in FDTD have been compared with safe guidelines given by the recognized body such as ANSI/IEEE, ICNIRP and Malaysia Communication and Multimedia Commission (MCMC).
- ii. A recommendation has been proposed for mobile-based stations and Wireless-LAN transmitters to ensure the safety of IIUM students and staff.

1.5. THESIS OUTLINE

The thesis chapters are organized as follows:

Chapter two: *Literature review*. This chapter has detailed Maxwell's equations and decribed electromagnetic wave propagations. Biological effects of electromagnetic fields, Specifc Absorption Rate (SAR), and other relevant details were explained. In this chapter, the standardization and limits of the EM wave and SAR were highligheted by authorized body

Chapter three: *Develop of Flat Phantom Model for Electromagnetic Radiation*. This chapter has elaborated on developing an analytical flat phantom model of human body. Here, mathematic calculations, boundary conditions and computational considerations are discussed in details. Chapter four: *Experimental setup and Radiation measurement*. To measure the electric field and power density from Wireless-LAN and mobile phone based stations, the Portable field measurement system which consists of tri-axis probe, portable spectrum analyzer and the data logging software for storing in computer, Rohde and Schwarz (2003) has been described.

Chapter five: Results and Analysis. In this chapter, the simulated electric fields and Local Peak SAR values from XFDTD6.4 Remcom Inc software and developed Matlab Codes have been compared with safe guidelines given by recognized body such as FCC (1997), ICNIRP (1998), ANSI C95.1 (1982), IEEE (1998), and other recognized bodies.

Chapter six: Conclusion. This chapter has concluded the finding of research.

CHAPTER 2

ANALYSIS OF ELECTROMAGNETIC FIELD RADIATION

2.1 INTRODUCTION

An extremely challenging problem is the prediction of the penetration of electromagnetic fields (EMFs) into a human body. The structure of the body is quite complicated, and the constitutive parameters vary with position (Kunz K. S., Luebbers R. J., 1993). The Finite Difference Time Domain (FDTD) method is used for investigating the interaction between the human body model and EM waves.

The Finite Difference Time Domain (FDTD) approach has been existed since the 1960's but has gained great popularity in recent years with the increased performance from computers. While many electromagnetic simulation techniques are applied in the frequency-domain, the Finite Difference Time Domain (FDTD) method solves Maxwell's equations in the time domain. This means that the calculation of the electromagnetic field values progresses at discrete steps in time.

One benefit of the time domain approach is that it gives broadband output from a single execution of the program. However the main reason for using the Finite Difference Time Domain (FDTD) approach is the excellent scaling performance of the method as the problem size grows. Moreover FDTD method provides direct solution to Maxwell's equations without much complexity and also it takes into account both electric and magnetic fields in 3D model (Buchanan W. J., 1996). As the number of unknown increases, the FDTD approach quickly outpaces other methods in efficiency. The Finite Difference Time Domain (FDTD) has also been identified as the preferred method for performing electromagnetic simulations for biological effects from wireless devices. The XFDTD method has been shown to be the most efficient approach and provides accurate results of the field penetration into biological tissues (Remcom, Inc., 2006).

2.2. ELECTROMAGNETIC FIELDS THEORY

Electromagnetic (EM) fields are ubiquitous in today's environment, being inherent to communications, power and other needs of modern society. The proliferation in the use of E-M fields has been accompanied by an increased concern regarding their safety. Electric (\vec{E}) and magnetic (\vec{H}) fields are produced by electric charges and their motion. Their behavior and relationship are described by the set of equations known as Maxwell's equations.

A time-varying electric field produces a magnetic field and vice versa, (Clements C. D., 2004). An electric field may be described by its magnitude, \vec{E} (Vm^{-1}), and the electric flux density, \vec{D} (*coulomb* m^{-2}). These quantities are related through the electrical properties of the medium described by the permittivity ε :

$$\vec{D} = \mathcal{E}\vec{E} \tag{2.1}$$

A magnetic field is described by its magnitude, \vec{H} ($A m^{-1}$), and the magnetic flux density, \vec{B} (*Tesla*(*T*)). These are related through the magnetic properties of the medium described by its permeability μ :

$$\bar{B} = \mu H \tag{2.2}$$

2.2.1. Maxwell's Theory

To understand the large-scale of electromagnetic phenomena one should consider the laws which govern the electric and magnetic fields. In his lifetime, Maxwell had collected the information available and showed that they can be expressed by the following four laws known as fundamental Maxwell's theory (Ghosh S. N., 2002).

2.2.1.1 Faraday's Law of induced Electromotive Force

It states that the e.m.f. Induced in a closed path is equal to the rate of change of magnetic flux linking the path. Therefore, the following relation is derived:

$$emf = \oint \vec{E} \cdot d\vec{l} = -\frac{\partial \phi}{\partial t}$$
(2.3)

The integration is carried over a closed path where in the magnetic flux linking it is given by ϕ .

2.2.1.2 Ampere Current Law

The line integral of the magnetic field around a closed path is equal to the current linking the path. Then, it can be written as follow:

$$\oint \vec{H} \cdot d\vec{l} = i \tag{2.4}$$

2.2.1.3 Gauss's Law for Electric field

It states that the net *outward* electric flux through any closed surface is equal to the charge enclosed by the surface. Hence:

$$\oint \vec{D} \cdot d\vec{s} = q \tag{2.5}$$