INTERNATIONAL ISLAMIC UNIVERSITY MALAYSIA

ACHIEVEMENT IN PROBLEM SOLVING AND METACOGNITIVE THINKING STRATEGIES AMONG UNDERGRADUATE CALCULUS STUDENTS

BY

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ABSTRACT

The purpose of this study was to investigate problem solving in the field of calculus. The study investigated the metacognitive thinking strategies employed by lecturers that were considered as subject matter specialist. The study developed and operationalized a metacognitive thinking strategies model. This model was then tested for its reliability and its predictive nature towards problem solving skills in non-routine calculus problems. A questionnaire was then administered among 480 first year undergraduate students who were selected randomly. The rate of return was about 90%. Using principal component analysis (PCA) the study successfully identified seven underlying dimensions of metacognitive thinking strategies. They are Self-efficacy, Define, Explore, Accommodate, Strategize, Execute and Verify. Finally, the researcher applied multiple regression analysis to evaluate the predictive ability of the identified predictor and the performance on routine and non-routine calculus problems. The study found that problem solving skills is acquired through practice and utilization of thinking strategies which is the corner stone on which advance mathematical ideas and particularly calculus are build on. This study revealed that there are six meaningfully predictive factors of calculus problem solving performance. It found that "strategize" is the major predictive of calculus problem solving performance, followed by "accommodate, self-efficacy, define, explore and then execute". Further analysis revealed that Strategies, Accommodate and Self-Efficacy were considered most significant with substantial practical importance. With these findings, educators will be able to clinically evaluate a person's ability to regulate, monitor and control his or her own cognitive processes. Instructional strategies can then be developed for those individuals having difficulty functioning in the learning environment.

ملخص البحث

استهدفت هذه الدراسة استقصاء حل ا شـكلات أو ا سـائل حسـاب التفاضـل. استقصت الدراسة استراتيجيات التفك فيما وراء الإدراك، ال اسـتخدمها ا درسـون الأوائل الذين اعت وا متخصص هذه ا ادة. أعدت الدراسة وذجـا لاسـتراتيجيات التفك فيما وراء الإدراك وجهزته للعمل. وقد اختبار هذا النموذج من حيث درجـة ثباته وطبيعته التنبؤية اه سلوك الطلاب حل مشكلات أو مسائل حساب التفاضل غ الروتينية. إجراء الاستبيان على 480 طالبا من طلاب السنة الأو ا امعية، اخـت وا عشوائيا. وقد أعيد حوا 90 % من الاستبيانات بعد الإجابة عنها. وباستعمال ليـل ا كونات الرئيس (PCA) التعرف بنجاح على سبعة أبعاد أساسـية لاسـتراتيجيات التفك فيما وراء الإدراك ؛ هي الاعتداد بالنفس ، والتوضيح، والاستطلاع، والتكيـف ، والتخطيط، والتنفيذ، والإثبات. وأخ ا ، استخدم الباحث منهج التحليل الارتدادي ا تعدد لتقييم القدرة التنبؤية للمتنبئات ال التعرف عليها، وأداء الطلاب حل مشكلات أو مسائل حساب التفاضل الروتينية وغ الروتينية.ولقد أشارت الدراسـة إ أن سـلوك الطلاب حل مسائل حساب التفاضل يتم اكتسابه عن طريق التدريب ، والاستفادة من استراتيجيات التفك ال هي حجر الأساس الذي تب عليه أفكـار علـوم الرياضـيات ا تقدمة وعلى وجه ا صوص حساب التفاضل. ولقد أسفرت الدراسة عن أن هناك ستة عوامل تنبؤية للقيام ل مسائل حساب التفاضل، فقد وجـد أن التخطـيط أو وضـع استراتيجيات هو العامل التنبؤي الرئيس للقيام ل مسائل حسـاب التفاضـل، يتلـوه التكيف، والاعتداد بالنفس، والتوضيح، والاستطلاع، وبعدها التنفيذ.كما أسفر التحليـل الإضا عن أن التخطيط أو وضع استراتيجيات، والتكيف، والاعتداد بالنفس ، كلها تعت ذات أ ية بالغة ا ا من أ ية عملية جوهرية. وبالإفادة من نتائج هذه الدراسة يصـبح التربويون قادرين على التحليل الإكلينيكي لقدرة فرد ما على تنظـيم ومراقبـة وضـبط

عملياته الإدراكية.وبذلك   كن تطوير استراتيجيات تدريس هؤلاء الذين   دون صـعوبة عند أدائهم   ا   يط التعليمي.

APPROVAL PAGE

The thesis of Logendra Stanley Ponniah has been examined and approved by the followingI:

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DECLARATION PAGE

I hereby declare that this dissertation is the result of my own investigations, except where otherwise stated. I also declare that it has not been previously or currently submitted as a whole for any other degrees at IIUM or other institutions.

Logendra Stanley Ponniah

Signature…………………………… Date………………………….........

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Affirmed by Logendra Stanley Ponniah

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Signature Date

DEDICATION

This Dissertation is dedicated to my grandfather and to my mother for their love, courage, support and sacrifice for me.

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CHAPTER ONE

INTRODUCTION

This chapter provides an introduction to and a general overview of the study. In the statement of the problem, the need for an investigation of students' metacognitive strategies in the field of calculus problem solving and the significance and limitations of the study are discussed. The research questions that are to be addressed in this study, the limitations and definitions of the key terms that are involved in this study, are then presented.

BACKGROUND

Metacognition

The term metacognition was first introduced in the literature on metamemory by Flavell, Friedrichs, and Hoyt (1970). They defined it as:

> knowledge concerning one's own cognitive processes or anything related to them, e.g., the learning-relevant properties of information or data ... Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of those processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete [problem-solving] goal or objective. (p. 232)

Metacognition then, is the ability to monitor, regulate and control any cognitive enterprise (Flavell, 1979). This construct is of interest to many researchers because it may be the key to, learning differences and learning difficulties experienced by many individuals (Montague, 1992; Wittrock, 1986; Wong, 1987). The ability to regulate, monitor and control the learning processes affects the way individuals utilize and implement strategies in many domains such as reading and mathematics (Montague, 1991; Wong, Wong & Blenkinsop, 1989). If individuals are not monitoring their thought processes, then, they will not be able to effectively learn and solve problems, particularly mathematical base problems.

Metacognition is essential for any extended activity, especially problem solving because, the problem solver needs to be aware of the current activity and of the overall goal, the strategies used to attain that goal, and the effectiveness of those strategies. The “mind” exercising Metacognition asks himself/herself: What am I doing? And how am I doing it? These self-directed questions are assumed in the application of all algorithms towards solving problem. However, in practice, teachers cannot simply assume that students will engage in metacognition; it must be taught explicitly as an integral component of problem solving.

Problem solving requires both the vigilant monitoring and the flexibility permitted by metacognition. When solving problems, the means adopted for the purpose shift continually depending on one's position relative to the desired goals. Even goals change as old goals are superseded by new and better ones. Maintaining flexibility is essential. Too often, we feel bound to a chosen strategy and continue to apply that strategy even if it leads us wildly astray. When this happens, it is usually wrong to conclude that we must start over. The important question always is, "What do I do now, given my goal, my current position, and the resources available to me?” Getting off course along the way is fully expected. Cool-headed reappraisal is the best response - not mindless persistency, panic, or surrender.

Problem Solving and Mathematics

Problem solving has been part of mathematics since the first human efforts to formalize mathematics. According to Chi, Glaser, and Rees (1982), the area is becoming a prominent field of research in cognitive psychology. There are substantial research findings that substantiate the claim that metacognition enhances problem-solving behavior among its pupils. According to Davidson & Sternberg, (1998) "Metacognition appears to function as a vital element contributing to successful problem solving by allowing an individual to identify and work strategically. This link between metacognition and success in mathematical problem solving through an interplay between cognitive and metacognitive behaviors is well documented in the literature (Artzt & Armour-Thomas 1992; Carr and Biddlecomb, 1998; Linn 1987; Quinto & Weener, 1983).

Before we can do it, talk about it, or teach it, we should discuss what problem solving is, and what it is not. Problem solving is a process that evolves through life. Problem solvers encounter situations that intrigue them enough to work through a mystery to arrive at a satisfactory solution. Problem solving makes use of previously acquired knowledge, skills, and comprehension, which are then synthesized into a new format that provides avenues to resolve the question at hand. The expectation is that problem solving is going to require the student to use acquired facts and information in the problem to solve the mathematical mystery in which they are currently engaged. Most people interviewed in this study think that problem solving can be taught. They believe that problem solving skills evolves out of the practice of solving problems.

It is difficult to discuss problem solving without giving respect to George Polya. Many consider Polya to be the father of modern thought on problem solving. In 1945 Polya wrote *How to Solve It*, which provides a wealth of information and includes a list of four problem-solving steps, which are:

1. *Understand* the problem.

2. Make a *plan* for solving the problem based on data and ideas given.

3. *Carry out* the plan.

4. *Look back* at the solution.

Comprehension, planning, implementation, and follow-up are basic steps involved in the business world. The similarities between these and Polya's list could be additional selling points for students. A review of the literature on problem solving will show a variety of lists of steps but, in almost every case, Polya's four steps form a basic framework. Those four steps are generic problem-solving skills that can be applied in a multitude of real-life settings.

Problem solving assumes specific importance in the constructivist's approach to teaching mathematics, which relies on the premise that knowledge is constructed by learners as they attempt to make sense of their own experiences. Students must become active participants, involving themselves in the total learning environment. If either the teacher or student falls short of their given responsibility, the net result will be, a less than satisfactory learning environment for mathematics.

We must be able to motivate all students to learn mathematics. No longer can we accept the idea that mathematics is only for the best and the brightest. Too many basic mathematical concepts are an integral part of our daily lives to permit such a position.

To change our students mindset and give them a willingness to accept mathematics, we need to understand the field. Some say mathematics is a way of thinking. This becomes evident to us as we do proofs, examine patterns, or organize our approaches to new and different problems. Others say mathematics is a language. We can understand that statement as we do talk in our special language of *X*s and *Y*s, graphs, patterns, and so on. However, if that language is spoken only by a selected few, what good is it? All students are capable of learning mathematics, and it is imperative that we include all of them in our inner circle of basic mathematical language. The inclusion should lead them to understand the world of basic mathematics and empower them to be more productive members of society.

STATEMENT OF PROBLEM

There is evidence in the literature and from personal experience that the teaching and learning of calculus is problematic. For instance Schwalbach and Dosemagen, (2000) confers that the problems of teaching calculus are extraordinarily difficult. They argue that there are "n" different topics, each of which must be treated before any of the others. You cannot do that when "n" is much greater than one. There is also the problem of how much rigor to build into the course. Schwalbach and Dosemagen further lament that, "as mathematicians, how do we hold up our heads amongst our colleagues and resist the pressure of the engineers down the hall to teach cookbook procedures."

Students have difficulty learning the methods and acquiring the levels of conceptualization needed to engage in efficient problem solving on calculus problems. For example after two years of undergoing high school calculus, most students are unable to relate and most of all apply what they have learned.

> Research into students' understanding of the central processes of Calculus - differentiation and integration has shown difficulties. Students have a strong tendency to reduce mathematics of this topic to a collection of algebraic problems, while avoiding graphics as well as geometrical images. (Dreyfus, 1992; p. 34)

Schoenfeld (1985a) explains how solving Calculus problems requires a sophisticated knowledge of mathematics and 'a substantial amount of thinking.' In an earlier paper Schoenfeld (1982,) had commented that:

> Roughly half of our students see Calculus as their last mathematics course. Most of these students will never apply Calculus in any meaningful way (if at all) in their studies, or in their lives ... the only reason they can perform with any degree of competency on their

> final exams is that the problems on the exams are nearly carbon copies of problems they have seen before; the students are not asked to think, but merely to apply well-rehearsed schema for specific kinds of tasks. (p.192)

This does not sound good, particularly for those who are going to be using Calculus in their lives in various vocations. Schoenfeld says, it is the sophistication and abstractness of the subject, which makes it difficult to teach Calculus, and for the students to acquire a conceptual understanding of it (Schoenfeld, 1985a).

The field of calculus is not only problematic to teach and to learn, it is also problematic for research. It covers a large area of topics including limits, functions, differentiation, integration and differential equation just to name a few. Researchers have also approached this in a variety of fashions; such as problem solving, conceptual understanding, spatial visualization, pedagogy, and technology application in mathematical modeling, just to name a few. It was through analyzing the assortment of fields such as that, the researcher decided on choosing differentiation as the content domain of the study. Furthermore, differentiation is one of the first ideas that is introduced in the realm of calculus that has a great deal of application. The failure to understand differentiation will certainly hinder students' mastery of subsequent ideas and skills.

The review highlights what the students do wrong and when they might do it more frequently. It also demonstrates how problem solving is better facilitated in certain contexts and how we might acquire more information about students' mathematical thought processes. What is missing is the answer to why students do what they do and whether certain cognitive techniques can be implemented into effective and ineffective behavior. It is therefore the aim of this thesis to understand more about the thinking strategies students employ when they attempt to solve non routine or novel calculus problems. This would enable researchers to understand more about mathematical thinking and that teachers might be better equipped to teach students a deeper and more conceptual understanding of calculus in the future.

In the review of similar studies, the researcher found that several studies examined the correlation between metacognition thinking strategies with problem solving skills or mastery of certain bodies of knowledge. There is a recognizable trend within these studies. Those studies that use qualitative methodology generally tend to indicate a significant relationship between metacognitive behavior and problem solving ability. On the other hand, quantitative studies which generally employed some kind of quasi-experimental designs tend to show insignificant relationships.

It is the researcher's opinion that metacognitive behavior cannot be disseminated within a short period of time. Furthermore, these studies do not clearly indicate whether metacognition was introduced within the context of knowledge of the course. The evidences indicate that these researchers presented the notion of metacognition independent of the domain of knowledge. If this is true, then it is left to the students to embody the metacognition thinking strategies to the body of knowledge. This obviously, introduces the element of chance. Summarily, these researchers generally used some kind of statistics to establish causal relationships between metacognition and problem solving.

The final point that the researcher would like to raise is that there is a lack of scale to measure metacognitive thinking strategies within the domain of calculus. Some of the studies covered in the literature review used the scales developed by Schoenfeld (1983) and Sternberg (1986). Unfortunately, these instruments seem too general to distinguish the polarity of thought processes. For instance, some studies used questionnaires that were too general. Respondents were responding to items with no particular reference points. Whereas such general instruments may well be workable in other fields but, problem solving in calculus is very demanding and the researcher feels that it requires a rather specific instrument. Any lack of precision may have adverse effects on the respondents' ability to answer coherently.

PURPOSE FOR THE STUDY

This study had two objectives. On the one hand, it attempts to investigate the perception of lecturers of calculus and their reflection on the importance of problem solving skills in calculus. At the same time, the researcher solicited the thinking strategies that these experts use, when they are forced with a novel situation. The second purpose of this study is to develop an instrument to adequately identify metacognitive strategies utilized by individuals' in the processes of solving mathematical problems. This study, is particularly interested in measuring metacognitive strategies used by first year undergraduate students. It attempts to explore some potential correlation between the acquisition of metacognitive strategies and the actual mathematical problem solving skills. The study attempted to generate evidence of the validity of scores on an instrument designed to evaluate levels of metacognition, so that, an accurate measure of metacognition can be made for diagnosis, preparing remedial activities, and for further research. More precisely, the psychometric properties of an instrument designed to measure metacognition for problem solving will be developed, analyzed and validated.

Within this thesis, the researcher would like to quantitatively validate the notion that metacognitive behavior does enhance the problem-solving ability. The principal objective of this thesis is to come up with a psychometrically sound self-