



AN ANALYSIS OF AL-KHWĀRIZMĪ'S
CONTRIBUTION TO MATHEMATICAL SCIENCES
WITH A SPECIFIC REFERENCE TO ALGEBRAIC
SCIENCE

BY

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ABSTRACT

This research studies the original Arabic version of Al-Khwārizmī's *Al- Mukhtasar fī HisÉb Al-Jabr wa'l-MuqÉbala* and explanations of his work on Arithmetic *KitÉb Al-Jem wa'l TafrÉq bi HisÉb Al-Hind* which is available only in Latin under the title of *Algoritmi de numero Indorum*. This work has used secondary sources in the form of books, articles, and internet documentations. The methodology employed in this study is a qualitative analysis of the collected data. The primary focus of the research has been on Al-Khwārizmī's overall contribution to mathematical sciences which eventually helped in the development of all branches of mathematics in the West. Although traditionally it has been known that algebra is an Arabic word and the use of the Arabic numerals originated through these studies, historians of mathematics have discovered that Muslim knowledge of mathematical schemes during the Medieval Times contributed to a great extent to the Renaissance in Europe. Al-Khwārizmī's algebra is already included in contemporary educational curricula in universities and secondary schools, therefore, Al-Khwārizmī's contribution to mathematics made him number one amongst the most celebrated mathematicians in the history of mankind. This study relates the pre-Islamic origins of mathematics and its development as an integral part during the Islamic civilization culminating with the creation of the academy of science called *Bait Al-Hikmah*. This study concludes by highlighting the essential principles Muslim should adapt to bridge the gap that separated between the Muslims' civilizational legacy and the present scientific and technological advancement of the West.

ملخص البحث

هذا البحث يدرس النسخة العربية الأصلية لكتاب الخوارزمي في علم الجبر كتاب المختصر في حساب الجبر و المقابلة، و شروحات لعمله علي الحساب كتاب الجمع والتفريق بحساب الهند، حيث أن هذا الكتاب فقدت نسخته العربية الأصلية و لكنه متاح باللغة اللاتينية تحت عنوان Algoritmi de Nemero Indorum أيضا يعتمد هذا البحث علي مصادر ثانوية للبيانات تشمل الكتب والمقالات العلمية، و وثائق الإنترنت. إن المنهجية المستخدمة في هذا البحث العلمي تمثلت في التحليل النوعي للبيانات التي تم جمعها. لذلك فإنها تركز بصورة أساسية علي توثيق مساهمة الخوارزمي الشاملة للعلوم الرياضية وما قامت به من دور عظيم ساهم في تطوير كل فروع العلم في الغرب و كلمة "الجبر" - كما هو معروف الآن - هي كلمة عربية، والأرقام المستخدمة في الغرب الآن هي الأرقام العربية. و لقد عرف و تعلم المؤرخون في الأيام القليلة الماضية ما أنجزه المسلمون في العصور الوسطى، و أثار هذه المعرفة علي النهضة الحديثة في أوروبا. إن مساهمة الخوارزمي في علم الجبر شملت الرياضيات المعاصرة الداخلة في جميع المناهج العلمية في الجامعات و المدارس في يومنا الحاضر. لذلك فإن مساهمة الخوارزمي للرياضيات جعلته يقف في الصف الأول من صفوف الرياضيين في تاريخ الإنسانية. بحثت هذه الدراسة في الخلفية التاريخية للرياضيات قبل الإسلام، وتطورها كجزء لا يتجزأ من الحضارة الإسلامية وبلوغ ذروتها مع إنشاء أكاديمية العلوم أو ما يسمى بيت الحكمة. وتخلص الدراسة بتسليط الضوء على المبادئ الأساسية التي ينبغي للمسلم أن يتبناها لسد الفجوة التي تفصل بين هذا الموروث الحضاري المتنوع والتقدم العلمي والتكنولوجي الحالي للغرب.

APPROVAL PAGE

I certify that I have supervised and read this study and that in my opinion; it conforms to acceptable standard of scholarly presentation and is fully adequate, in scope and quality, as a thesis for the degree of Master of Arts (Islamic and other civilization)

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DECLARATION

I hereby declare that this dissertation is the result of my own investigations, except where otherwise stated. I also declare that it has not been previously or concurrently submitted as a whole for any other degree at IIUM or other Institutions.

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**AN ANALYSIS OF AL- KHWĀRIZMĪ'S CONTRIBUTION TO
MATHEMATICAL SCIENCES WITH A SPECIFIC REFERENCE TO
ALGEBRAIC SCIENCE**

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TABLE OF CONTENTS

Abstract	ii
Abstract in Arabic	iii
Approval Page.....	iv
Declaration Page	v
Copyright Page.....	vi
Acknowledgements.....	vii
CHAPTER ONE: INTRODUCTION	1
1.1 Introduction	1
1.2 Al-Khwārizmī’s Life And Works	4
1.3 The Objectives Of The Study.....	9
1.4 Research Questions	10
1.5 The Significance Of The Study.....	11
1.6 Research Methodology.....	12
1.7 The Expected Contribution Of The Study	12
1.8 Literature Review	13
CHAPTER TWO: HISTORICAL BACKGROUND	21
2.1 Prehistorical Mathematical Sciences.....	21
2.1.1 Introduction	21
2.1.2 Mathematics in Ancient Mesopotamia	26
2.1.2.1 Primary Materials of Ancient Mesopotamia.....	27
2.1.2.2 Number Order and Arithmetic Operations in Mesopotamia.....	27
2.1.2.3 Algebra and Geometric Problems in Ancient Babylonian.....	29
2.1.3 Ancient Egyptians’ Achievements in Mathematics	32
2.1.3.1 Primary Materials of Ancient Egyptians.....	33
2.1.3.2 Numerals and Types of Writing.....	34
2.1.3.3 Arithmetic Operation	37
2.1.3.4 Quadratic Equations in Egyptian Civilization	38
2.1.3.5 Geometric Problems and Measurements.....	40
2.1.4 The Ancient Greek Mathematics	47
2.1.4.1 Properties of Greek Mathematics.....	48
2.1.4.2 Greek Number and Arithmetic.....	50
2.1.4.3 How Greek Numbers worked	51
2.1.4.4 Greek Geometric Algebra	52
2.1.4.5 The Most Famous Mathematics in Ancient Greece.....	55
2.1.5 Ancient Roman Mathematics.....	73
2.1.5.1 The Influence of Greek Mathematics on Roman Mathematics.....	75
2.1.5.2 Numeration and Computation in the Roman Era.....	78
2.1.6 Hellenistic Mathematical Sciences	79
2.1.6.1 Archimedes (287- 212 B. C.).....	80
2.1.6.2 Apolonius of Perga (240- 190 B.C.)	81
2.1.7 Indian Mathematical Sciences.....	83

2.1.7.1 History of Zero.....	88
2.1.8 Chinese Contribution to the History of Mathematics	91
2.2 Original Sources Of Al-Khwārizmī’s Algebra	94
CHAPTER THREE: THE HOUSE OF WISDOM	100
3.1 The Role Of Bait Al-Hikmah In Producing New Knowledge	100
3.2 Al-Khwārizmī’s Contribution To Algebra.....	109
3.3 The Effect Of Al-Khwārizmī’s Works On His Immediate Enviroment	119
3.3.1 Thabit Ibn Qurra (826- 901 A.D).....	121
3.3.1.1 The Works of Thabit Ibn Qurra	122
3.3.2 Abu Kamil (850- 930 A. D.).....	125
3.3.3 Omar Al-Khayyám (1050- 1123 A. D.).....	130
3.4 Al-Khwārizmī’s Impact Contributes To The Development Of Mathematics In Europe	139
3.5 Al-Khwārizmī’s Algebra And The Modern Mathematics As Regular School Courses.....	144
3.5.1 Some Examples of Simultaneous Equations.....	149
3.5.2 Problem Solved By Quadratic Equation	151
3.5.3 Mensuration	151
CHAPTER FOUR: OVERALL IMPACT OF AL-KHWĀRIZMĪ’S CONTRIBUTION TO MATHEMATICS.....	162
4.1 Al-Khwārizmī’s Contribution To Mathematics Contributed To The Growth Of Civilization	162
4.2 Reasons Behind Stagnation Of Mathematical Sciences Under Muslim Rule	170
4.3 Transformation Of Al-Khwārizmī’s algebra Duringthe Last Few Centuries	186
CONCLUSION	195
BIBLIOGRAPHY	198
GLOSSARY.....	209

CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

In the book *The Making of Humanity*, Robert Briffault said,

Mathematics is the most momentous contribution of Arab civilization to the modern growth in which the decisive influence of Islamic culture is not traceable. Nowhere is it as clear and momentous as in the genesis of that power which constitutes the permanent distinctive force of the modern world, and the supreme source of its victory – natural science and the scientific spirit.¹

The most important contribution of Islamic mathematics is in the area of algebra. One of the earliest Islamic algebra texts was written in about 825 A. D. by Al-Khwārizmī and was entitled *Al-Mukhtasar fī Hisāb al-Jabr wa al-Muqābala* (the condensed book on the calculation of algebra and arithmetic). The other work by Al-Khwārizmī on arithmetic is the oldest known Arabic book on calculation with Indian-Arabic numerals.² This book has survived in a Latin translation but the original Arabic version was lost. The translation was most likely done in the 12th century by Adlere of Bath (he was a medieval mathematician and natural philosopher, and a crucial figure in the development of early European thought. He is known both for his original works and translation of many important Greek and Arabic scientific works of astrology and mathematics). The Latin translation is untitled, but is commonly referred to by the first two words with which it starts: *Dixi algorizmi* (so said Al-

¹ Robert Briffault, *The Making of Humanity* (London: George Allen & Unwin L T D. Ruskin House, 40 Museum Street W. C. I., 1919), 190.

² Victor J. Katz, *A History of Mathematics: An Introduction* (3rd edn.) (Chicago: Pearson Education, Inc., 2009), 271.

Khwārizmī).³ However, the book entitled *Al-Mukhtasar fī Hisāb Al-Jabr wa'l-Muqābala* was more influential than his arithmetical work because the book is devoted to finding solutions to practical problems, which that were facing the communities in those days. Such as matters of inheritance, legacies, partition, shares, lawsuits, and commerce, with over eight hundred cases. Al-Khwārizmī took materials previously developed by the Babylonians, combined it with the classical Greek heritage of geometry, and produced a new science called algebra and developed it further. He is the first mathematician who differentiated between algebra and geometry, and he also presented graphic and geometrical solutions of linear and quadratic equations.

Al-Khwārizmī's contribution to algebra affected the development of all branches of mathematics positively. Al-Khwārizmī had two treatises on arithmetic and algebra in the entire world. The works of Al-Khwārizmī today are used in universities and schools as regular courses in the subject of algebra. For example, in most parts of the world today, the treatise on the arithmetic is utilized in teaching kindergarten and primary students how they can write numbers in the style that Al-Khwārizmī invented, and how they can use them for calculation. Moreover, the treatise on arithmetic is considered as the foundation of the modern technology industry and is known as algorithms treatise. This word is derived from the name of Al-Khwārizmī and this derivation may soon be forgotten because academicians and scholars use this word only as a reference to the arithmetical processes (addition, subtraction, division, and multiplication) and description of arithmetic text.

The other treatise *Al-Mukhtasar fī Hisāb al-Jabr wa Al-Muqābala* (the compendious on calculation by completion and balancing) has contents of the algebra

³ “Abū ‘Abdallāh Muhammad ibn Mūsā Al-Khwārizmī,” *Megagroup*, <<http://en.hamrohonim.net/ab-abdallah-muammad-ibn-m>>.

text. The first part of the book consists of a discussion of the rules that Al-Khwārizmī used to solve different types of linear and quadratic equations. These extracted rules are considered fundamentals of the modern mathematics, and are being taught in the educational institutions of the contemporary world. However, it is important to realize that the book was planned to be practical, and that algebra was introduced to solve problems of everyday life of his time.

Unfortunately, scholars and academicians failed to adequately acknowledge the important role of Al-Khwārizmī's algebra. In his book, in the preface of the *Episodes in the Mathematics of Medieval Islam*, J. L. Berggren said that,

Many people today know something of the debt our mathematics owes medieval Islamic civilization. They know algebra is an Arabic word and they speak of our Arabic numerals. In recent years, as well, historians of mathematics have re-learned what our medieval and renaissance forebears knew: the Islamic contribution affected the development of all branches of mathematics in the west and was of prime importance in many. Despite this no text book on the history of mathematics in English deals with Islamic contributions in more than a general way.⁴

Furthermore, in his book *Men of Mathematics* E. T. Bell wrote,

It may be doing our predecessors an injustice to emphasize modern mathematical thought with but little reference to pioneers who took the first and possibly the most difficult steps. But nearly everything useful that was done in mathematics before the seventeenth century has suffered one of two fates: either it has been so greatly simplified that it is now part of every regular school course, or it was long since absorbed as a detail in work of greater generality.⁵

Among those mathematics pioneers was Al-Khwārizmī, the inventor of algebra. Therefore, this study aims to examine the pre-Islamic origins of mathematics and the original sources of Al-Khwārizmī's algebra. We will analyze how Al-Khwārizmī's contributions to algebraic science was transformed throughout the following few centuries and affected the development of all the branches of

⁴ J.L. Berggren, *Episodes in the Mathematics of Medieval Islam* (New York: Springer-Verlag, 1986), vii.

⁵ E. T. Bell, *Men Of Mathematics* (New York: Simon & Schuster, Inc., 1937), 14.

mathematics in the West and was of prime importance in many Western countries. This study also covers the important role of *Bait al-Hikma* (House of Wisdom), a school of translation House of Wisdom, the connection between Al-Khwārizmī's algebraic science and modern algebra, the reason behind stagnation of the mathematical sciences under Muslim rule, and how can Muslim regain their excellence in mathematical sciences.

1.2 AL-KHWĀRIZMĪ'S LIFE AND WORKS

In his book *A History of Algebra*, B. L. Van der Waerden states that a great deal of the life and the work of Al-Khwārizmī were covered by G.J. Toomer in volume V11 of the dictionary of scientific biography. So he writes,

There are not many details of Al-Khwārizmī's life that can be gathered from some of the Islamic notices in bibliographical works and different comments by Islamic historians and geographers. The epithet Al-Khwārizmī points to that he came from Khwarizm (Khorezm, conformity and corresponding to the modern Khiva and the area is south of the Aral Sea in the central Asia). But the historian Al-Tabri gives him the additional epithet 'Al-Qutrubbulli', indicating that he came from Qutrubbulli, a district between the Tigris and Euphrates not far from Baghdad, so perhaps his ancestors, rather than he himself, came from Khwarizm; this interpretation is confirmed by some sources which state that his "stock" (asl) was from Khwarizm.⁶

Abu Ja'far Muhammad Ibn Musa Al-Khwārizmī was born in 194 A. H. (780 A. D). Although his name indicates that his family was originally from Khwarizm, a region close to the Aral Sea, historians believe that Al-Khwārizmī was born in the city of Baghdad in the present day Iraq. Little is known about his personal life. Al-Khwārizmī's works and contributions to mathematics are still available and have survived in the original form. However, there is one original book on arithmetic that is not available in its original language. This book, entitled *Algoritmi de numero*

⁶ B.L.Van der Waerden, *A History of Algebra: From al-Khwārizmī to Emmy Noether* (Berlin: springer-Verlag, 1985), 3.

Indorum, is available in Latin language, and it is available only where it was translated, the library of the University of Cambridge, but Arab references also cites the missing treatise.⁷ There is no idea about how and where the original version on arithmetic was lost. Scientists, historians, and mathematicians have no comments where and how this book was lost, This book existed before the middle of the 13th century because it was translated during this period by some Western mathematicians such as Jurdanus de Nemore, who was one of the most important writers on mechanics in Latin to be compared with Leonardo Fibonacci and Nicole Oresme according to *Writing the History of Mathematics*. To the researcher's knowledge, this version and other books might have got lost when the Mongols invaded the Islamic heartland and destroyed or burned libraries and collection of books.

Abu Abdullah Ibn Musa Al-Khwārizmī (780-847 A. D.), the earliest mathematician of the Islamic world, was also the founder of a new algebra science. Many books about the philosophy of mathematics and the history of mathematics refer to his unique studies. Al-Khwārizmī worked in the ninth century under the patronage of the Caliph Al-Ma'mun in Baghdad. He became a member of Dar-ul Hikmah, which means the House of Wisdom, a kind of academy of scientists, which was founded in Baghdad most probably by Caliph Harun al-Rhasid. But it owes its prestige to Al Ma'mun, a great patron of knowledge and scientific study. At the time of Al Ma'mun, not only the science of mathematics that flourished, but other sciences as well.⁸

Muhammad Ibn Musa Al-Khwārizmī was the author of the greatest mathematical works. His most important books were on algebra and arithmetic, The

⁷ Conger, Heather, Shawn Overpay & Jimmy Scorer, *Al-Khwārizmī the Father of Algebra* (Upper Saddle River, NJ: Prentice-Hall Inc, 1995),
<https://www2.bc.edu/christian-zorn/work_and_research/hist_algebra.pdf> (accessed 5 May 2001).

⁸ Adnan Baki, "Al-Khwārizmī's Contribution to the Science of Mathematics: Al kitab Al Gabr Wa'l Muqabalah", *Journal of Islamic Academy of Sciences* (11 Woburn, Sq, Weich ONS London, U.K.), Vol. 10, no. 3 (1992): 225. <Medicaljournal-ias.Org/.../Belge/BakiDYENRPLU79710pdf>.

Persian Muslim Mathematician became very popular and a reference of mathematicians along the ages. The world of modern mathematics is familiar with the term algebra, which is derived from the Arabic word *Al-jabr* which means “meeting” or “relationship.”⁹ Algebra is considered a major branch of the mathematical sciences, and it is a regular course of primary and secondary schools at all levels, as well as in the university.

In Baghdad, in the 9th century, under the auspices of Al-Ma'mun, flourished the House of Wisdom. It was similar to the idea of the great Museum of Alexandria. At the House of Wisdom the translation of the great Hellenistic mathematical compositions was done by some scholars. Among these scholars was 'Abu Ja'far Muhammad Ibn Musa Al-Khwārizmī. He was the most proficient mathematician in the academic center, and wrote on different fields, including the numbering system of the Indians. A Latin translation of Al-Khwārizmī of the Indian numbers became one of the most major references in Europe, where a new system of numeration was introduced.¹⁰

Around 825 A.D., Al-Khwārizmī wrote the condensed book *Restoration and Reduction*. The title *Restoration and Reduction* refers to the process of adding or subtracting terms to eliminate them from one side of the equations.¹¹ Thus, Roger Cooke wrote that the word restoration here refers to the operation of keeping an equation in balance by transferring a term from one side to the opposite side with the

⁹ Help University, *Islamic Historical: the Phenomenal Book of the Great Mathematician*, <Islamichistorical.blogspot.com/-/algebra-book-phenomenal-work-of...> (Accessed 19 July, 2011).

¹⁰ Suzuki Jeff, *Mathematical in Historical Context* (Washington: The Mathematical Association of America, 2009), 86.

¹¹ *Ibid.*, 88.

opposite sign. The word reduction refers to the cancellation of like terms or factors from the two sides of an equation.¹²

Roger Cooke, in his book *The History of Mathematics* mentioned that the integration of intellectual interests with religious goodness that we saw in the case of the Indians is a trait possessed by Muslims. Al-Khwārizmī produced his algebra book with a hymn of praise of Allah, and then dedicated his book to al-Ma'mun:

That the fondness for science, by which God has distinguished the Imam Al-Ma'mun, the commander of the faithful, that sociability and respect which he shows to the learned, that promptitude with which he protects and supports them in the elucidation of obscurities and in the removal of difficulties, has encouraged me to compose a short work on calculating (the rules of) completion and reduction, confining it to what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, law-suits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, Geometrical computation, and other objects of various sorts.¹³

The curiosity created by the Quranic revelation made the early Muslim intellectuals practice creative thinking in its true sense. This is obviously mirrored in their original works and their contributions in translation and preservation of the scientific heritage of ancient sciences, such as Greek, Indian, and Persian.¹⁴ For example, Al-Khwārizmī's mathematical contribution was a practical textbook on algebra. Al-Khwārizmī's algebra book introduced solutions to the problems that were encountered in the early Islamic state concerning legacies, mensuration, and the daily transactions of Muslims. However, we should not forget the importance role of Al-Ma'mun as a patron and supporter of scientific research.

¹² Roger Cooke, *The History of Mathematics* (New Jersey: John Wiley & Sons, Inc., 2nd edn. 2005) 56.

¹³ *Ibid.*, 56.

¹⁴ Ibrahim A. Shogar, "The Scientific Thinking in Islam: Factors of Flourishing and Decline," *Revelation and Science*, Vol. 01, no. 02 (1433 H/ 2011), 1-13. <www.iium.edu.my/revival/index.php/revival/article/view/26/16>.

The first great advance in the Muslim world was the introduction of Arabic numerals. The numerals originated from India and were simplified, and this made the development of algebra possible. In Al-Khwārizmī's era, algebra resulted from a practical and imperial system to solve all kinds of problems and the issues of “inheritance, contracts, surveying, tax collection, legacies, partition, lawsuits, and trade, and all other dealings with one another. For example, the measuring of lands, the digging of canals, geometrical computations, and other objects of various sorts were concerned.¹⁵ So Al-Khwārizmī is considered the first mathematician who followed the scientific methodology in his studies because his studies were conducted with observation and experiment. He also differentiated between geometry and algebra and introduced algebraic and geometric solutions of quadratic equations in different ways.

Last of all, Al-Khwārizmī, the father of algebra, is a mathematician and also an astronomer, and a geographer. Al-Khwārizmī was selected as an important scientist in the science of the most prestigious in the academic center (*Bait Al-Hikmah*) of his day. This center was founded by Abbasid Caliphs in the intellectual city of the world, Baghdad. He was a genius scientist in the golden age of Islam in Baghdad, in the Abbasid Caliphate government's center. He was very instrumental and brought forward a lot of evidence in developing the sciences of algebra and arithmetic.¹⁶

Through his arithmetic, Al-Khwārizmī's name has become a common English word. With the title of his most important book *Al-Mukhtasar fī Hisāb Al-Jabr wa'l-Muqābala*, he has supplied us with an even more popular household term. The word

¹⁵ Ricia Elwell-Socci, *Middle Ages Technologies: Muslim Algebra* (Four River Character Public School), <http://fourriverscharter.org/projects/Inventions/pages/muslimworld_algebra.htm> (accessed Tuesday, 05 June, 2007).

¹⁶ Help University, *Islamic Historical: the Phenomenal Book of the Great Mathematician* <Islamichistorical.blogspot.com/-/algebra-book-phenomenal-work-of...> (Accessed 19 July, 2011).

algebra come from this title of this book Europe later learned this branch of mathematics bearing this name.¹⁷ Al-Khwārizmī wrote over six books of astronomical, mathematical works, and these works may be based on the Sind Hind derived from India. Apart from astronomical tables and treatise on the astrolabe and the sundial, Al-Khwārizmī wrote two books on arithmetic and algebra. These books played very influential roles in the history of mathematics.¹⁸ He also compiled a book called *Kitāb Surat Al-Ard* (Shape of the Earth). This work differed from Ptolemy's work, and he corrected Ptolemy's views in detail. It is a description of a world map and contains a list of the coordinates of the important places on it.¹⁹ They are a key stone in the modern technology industry in Europe, major references of the modern mathematical sciences.

1.3 THE OBJECTIVES OF THE STUDY

The current research will attempt to explore how Al-Khwārizmī's algebra science was transformed throughout the last few centuries, that affected the development of all branches of mathematics in the West and was of prime importance in many aspects. The researcher tries to find out the extent of the relationship between Al-Khwārizmī's algebraic science and modern algebra, how his contribution to mathematics helped to solve the problems that confronted the new Islamic state during the 9th century, and explain the extent Al-Khwārizmī contributions to algebra science has contributed to the growth of civilization. Therefore, this study aims to:

¹⁷ Carl B. Boyer, *A History of Mathematics* (New York: Princeton University Press, 1985), 251, 252.

¹⁸ *Ibid.*, 251.

¹⁹ Ibrahim B. Syed, "History of Islam: An Encyclopedia of Islamic History", <<http://historyofislam.com/contents/the-classical-period/al-Khwārizmī/>> (Accessed 10 November, 2008).

- Clarify the connection between the pre-Islamic origins of mathematics and the original sources of Al-Khwārizmī's algebra science.
- Elucidate the important role of the House of Wisdom (*Bait al-Hikma*) as an academic center.
- Explore the reasons behind stagnation of the mathematical sciences under Muslim rule.
- Explain the extent of Al-Khwārizmī's contribution to algebra is held growth to civilization.
- Highlight the relationship between Al-Khwārizmī's algebraic science and modern algebra.
- Survey the effect of Al-Khwārizmī's works on his immediate environment.
- Show how Europeans benefited from his work.
- Make it clear why Al-Khwārizmī's impact helped the development of mathematics in Europe while the advancement of mathematics declined in Islamic countries.

1.4 RESEARCH QUESTIONS

This thesis answers the following research questions:

- What are the pre-Islamic origins of mathematics and the original sources of Al-Khwārizmī's algebra?
- What role did the House of Wisdom (*Bait Al-Hikma*) play in history?
- How Al-Khwārizmī's contribution to mathematics was held in the growth of Islamic civilization?

- How Al-Khwārizmī's contributions to algebra science was transformed throughout the last centuries and how did they affect the development of all the branches of mathematics in the West, and were they of prime importance in many aspects?
- What is the relationship between Al-Khwārizmī's algebraic science and modern algebra?
- What are the reasons behind the stagnation of the mathematical sciences under Muslim rule?
- What is the effect of Al-Khwārizmī's works on his immediate environment?

1.5 THE SIGNIFICANCE OF THE STUDY

This study is significant in many ways it:

- Presents the original sources of Al-Khwārizmī's algebra.
- Highlights the great mathematical attainments by Al-Khwārizmī in the Islamic world during the 9th century A. D.
- Draws attention to the significance of algebra in the ninth century A. D. and its role in solving problems and challenges faced by the emerging states.
- Emphasizes the central role of the *Bait al-Hikma* (the House of Wisdom) in developing new knowledge.
- Explores to what extent Al-Khwārizmī's algebra science plays an important role in growth of civilization.
- Brings to light Al-Khwārizmī's impact European development.

- Gives details how Al-Khwārizmī's contributions to algebra science was transformed throughout the last few centuries and affected the development of all the branches of mathematics in the West and was of prime importance in many aspects.
- Highlights the connection between Al-Khwārizmī's algebraic science and modern algebra.
- Shows up the effect of Al-Khwārizmī's algebra on the people around him.
- Explains why Al-Khwārizmī's impact increased the mathematics development of the Europeans while there was a decline in the mathematics advancement in the Muslim countries.

1.6 RESEARCH METHODOLOGY

The research dealt with both the original Arabic version and translated texts of Al-Khwārizmī's *Al-Mukhtasar fī Hisāb Al-Jabr wa'l-Muqābala* (it is available in the original form). Explanations of his work on arithmetic (it is only available into Latin language entitled *Algoritmi de numero Indorum*) such as *Fundamentals of Algorithmic*, and *The Development of Arabic Mathematics: Between Arithmetic and Algebra* was also employed as the original version on arithmetic is only available in Latin. The secondary sources of the data include scholarly books, articles, and the internet documentations. The researcher used qualitative method to analyze the data collected.

1.7 THE EXPECTED CONTRIBUTION OF THE STUDY

The study is expected to illustrate the important role of Al-Khwārizmī's contributions to algebra science during the 9th century in encountering the needs and difficulties that

resulted from the emerging Islamic state. Likewise, this research emphasizes the original sources of Al-Khwārizmī's mathematics to produce a new algebra, Al-Khwārizmī's impact on Europe, and the crucial role of the House of Wisdom (*Bait Al-Hikma*). This research also aimed at explaining how Al-Khwārizmī's contribution to algebra science was transformed throughout the last few centuries and affected the development of all the branches of mathematics in the West and was of prime importance in many Western countries. It is also intended the relationship between Al-Khwārizmī's algebra science and modern algebra in order to discover the reasons behind the stagnation of mathematical sciences under Muslim rule, and finally discussed why Al-Khwārizmī's impact in mathematics influenced the development of Europe while it declined in Muslim countries.

1.8 LITERATURE REVIEW

After a thorough scrutiny of past research materials available, the researcher has concluded that there are no detailed studies on this subject. Therefore, the present research attempts to fill in the gap by investigating the significance of Al-Khwārizmī's contributions to algebra in modern as well as medieval eras. The following is a summary of the vital primary and secondary sources used in the analysis. Muhammad ibn Musa Al-Khwārizmī is one of the greatest scientific minds of the medieval period and was known as the 'father of algebra'. He wrote *The Kitāb Al-Jem wa'l Tafriq bi Hisāb Al-Hind* (The Book of Addition and Subtraction according to the Indian Calculation) also called *Kitāb Hisāb Al-adad Al-Hindî*. The original Arabic text was lost and only the Latin translations of the work are available. The translation was probably done in the twelfth century by Adler of Bath, Robert of Chester, Spanish Jew, John of Seville, and others. However, there is no evidence of when or why the

original Arabic text was lost. It should, however, be kept in mind that the original Arabic form of all the sources of the subject of arithmetic has been lost. Al-Khwārizmī's best known classical work on algebra is *Al-Mukhtasar fī Hisāb Al-Jabr wa'l-Muqābala* is available both in the original Arabic version and in the English language. This research primarily draws attention on this book. Other sources includes the history of mathematics, Islamic sciences during the 9th century, and makers of mathematics, how the Arabs transformed the Western Civilization, sources of the hidden debt to Islamic civilization as well as other sources that are available in electronic documentations.

Moreover, the original Arabic version of *the Al-Mukhtasar fī Hisāb Al-jabr wa'l-Muqābala* was written in 825 A. D., and translated into Latin in the 12th century A. D. The English version of *the Al-Mukhtasar fī Hisāb Al-jabr wa'l-Muqābala*, was translated by Fredric Rosen in 1831 A.D, and the last copy was printed in 1989 A.D. by Pakistan Hijra Council as a compilation of the two volumes in a single volume. . It is dedicated to discover solutions to practical problems which community met in their daily life about matters of inheritance, legacies, partition, shares, lawsuits, and commerce with over eight hundred examples. It contains a chapter called 'Logical Necessities in Mixed Equations from Ibn Turk's 'Algebra'. Al-Khwārizmī gave the rules for the solution of different kinds of linear and quadratic equations, which were supported in a number of cases by geometrical proofs. The first part of the book of *The Al-Mukhtasar fī Hisāb Al-Jabr wa'l-Muqābala* discusses the equations of the first and second degrees. The problems presented by Al-Khwārizmī were reduced to one of the six standard forms. He gave rules for the solution of each of the six forms and explained how to reduce any given problem to one of these standard forms by providing examples. The second part of the book dealt with practical measurements

by giving rules for deducing the area of various plane shapes including the circles, and for reasoning the volume of a number of solids such as cones and pyramids. The third part presents legacies with inheritance. This book is indispensable to a thorough analysis of Al-Khwārizmī's contribution as it contains his major mathematical ideas.

A History of Mathematics: An Introduction Victor J. Katz includes important topics that are beneficial to the present study; the book, to some extent, provides a world view of mathematics; ancient mathematics, medieval mathematics, early modern mathematics, and modern mathematics. It includes also Problems are taken from original sources; enabling students to understand how mathematicians in various times and places solved mathematical problems. It consists of four parts. The parts that are relevant to the present research are the first and second parts. The first part discusses the emergence of mathematics in the ancient world such as Egypt, Mesopotamia, the beginning of mathematics in Greek, and mathematical methods in the Hellenistic times. This part highlights the historical emergence of mathematics and was used in the analysis to explore the prehistoric origins of mathematics and original sources of Al-Khwārizmī's algebra science. The second part discusses medieval mathematics and parts of this section will be used to support the proposition on the importance of algebra science to encounter the emerging challenges of the state. This part contains topics such as mathematics in Islam, in India, in China, and mathematics in medieval Europe. This book was beneficial to the current study because it deals with Islamic mathematics.

Episodes in the Mathematics of Medieval Islam by J. L. Berggren, the last print was in 2003 A.D. and it contains six chapters. The author presents episodes from the mathematics of medieval Islam, and he describes the subject of the development of mathematics in its proper historical context, referring to specific Arabic texts. In