

CONVECTION FLOW OF MICROPOLAR FLUID OVER  
A FLAT SURFACE OF ANOTHER QUIESCENT FLUID

BY

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Sciences)

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## ABSTRACT

Micropolar fluid is well known due to numerous applications such as paint, blood, liquid crystal, silicon oil and human fluids. This fluid can be defined as a fluid that contained microstructure and capable to solve fluid phenomena involving microstructure that cannot be explained by classical Navier- Stokes equation. In practical, many situations are involving two fluids with difference density such as sea water intrusion, air flow on top of water and oil spill over water occur. Despite the significance, the literature produced are still limited. Therefore, this thesis is intended to fill the research gap. Consider a lighter density of micropolar fluid is impinging orthogonally on a stretching surface of another heavier density of micropolar fluid. To solve this problem, system of dimensionless governing equations which consists of continuity, momentum, angular momentum and energy undergo scaling analysis to become dimensionless system of governing equations. Later, the similarity transformation is used to obtain the system of nonlinear ordinary differential equations and solved using shooting technique with Runge - Kutta - Gill method. The algorithm is implemented in Jupyter Notebook using Python 3 language. It is found that the result is in very good agreement with the previous work. The numerical results acquired are velocity, microrotation, temperature, skin friction and Nusselt number. The results show that stretching surface enhances the velocity and microrotation of micropolar fluid for both upper and lower fluids. It is also found that, lower fluid has a greater boundary layer thickness compared to upper fluid.

## ABSTRACT ARABIC

### خلاصة البحث

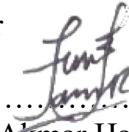
السائل القطبي الميكروسكوبي أو المايكروبولاري معروف لتطبيقاته العديدة، على سبيل المثال: في الدهانات، والدماء، والبلورات السائلة، وزيت السيليكون، والسوائل البشرية. يمكن تعريف هذا السائل على أنه سائل يحتوي على بنية مجهرية قادر على حل ظواهر السوائل المتضمنة لبنية مجهرية لا يمكن تفسيرها بواسطة معادلة نافير-ستوكس الكلاسيكية. من الناحية العملية هناك العديد من الحالات المتضمنة لسائلين بكثافتين مختلفتين، مثل تسرب المياه المالحة، وتدفق الهواء فوق الماء، والتسرب النفطي في الماء. على الرغم من أهمية هذا الموضوع فإن المؤلفات المتعلقة لا تزال محدودة، ولذلك هدفت هذه الدراسة إلى سد الفجوات البحثية. تم اعتبار اصطدام كثافة أخف للسائل الميكروبولاري بشكل متعامد مع سطح ممتد لسائل الميكروبولاري أعلى كثافة. لحل هذه المشكلة تم القيام بتحليل تحجيمي لنظام المعادلات الحاكمة اللابعدي المكون من الاستمرارية، والزخم، والزخم الزاوي، والطاقة وذلك لجعله نظاما لابعديا للمعادلات الحاكمة. بعدها تم استخدام تحويل التشابه للحصول على نظام معادلات تفاضلية عادية غير الخطية، وتم حلها بالرمزية باستخدام طريقة رانغ-كوتا-جيل. تم تنفيذ الخوارزميات ببرنامج جوبتر نوتبوك باستخدام لغة Python 3. كانت النتائج في اتفاق جيد جدا مع الأبحاث السابقة. النتائج العددية التي تم الحصول عليها هي: السرعة، ودرجة الحرارة الدقيقة، واحتكاك الجلد، ورقم نسلت (Nusselt). أظهرت النتائج أن تمديد السطح يعزز من السرعة والدوران الدقيق للسائل الميكروبولاري العلوي والسفلي، ووجد أيضًا أن السائل السفلي له طبقة حدية أكثر سماكة مقارنةً بالسائل العلوي.

## APPROVAL PAGE

I certify that I have supervised and read this study and that in my opinion, it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a thesis for the degree of Master of Science (Computational & Theoretical Sciences)



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## DECLARATION

I hereby declare that this thesis is the result of my own investigations, except where otherwise stated. I also declare that it has not been previously or concurrently submitted as a whole for any other degrees at IIUM or other institutions,

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## **ACKNOWLEDGEMENT**

Firstly, it is my utmost pleasure to dedicate this work to my supervisor and my family, who granted me the gift of their unwavering belief in my ability to accomplish this thesis: thank you for your support and patience.

I wish to express my appreciation and thanks to those who provided their time, effort and support for this project. To the members of my dissertation committee, thank you for sticking with me.

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## LIST OF SYMBOLS

$a$	Constant
$c$	Constant
Gr	Grashof number
$j$	Microinertia density
$k$	Thermal conductivity
$K$	Micropolar parameter
$n$	Concentration
$N$	Microrotation
Pr	Prandtl number
Ri	Richardson number
$T$	Temperature
$u$	Velocity of fluid along $x$ -axis
$v$	Velocity of fluid along $y$ -axis
$\alpha$	Suction parameter for upper fluid
$\beta$	Suction parameter for lower fluid
$\gamma$	Spin gradient viscosity
$\kappa$	Vortex viscosity
$\rho$	Density of micropolar fluid
$\mu$	Dynamic viscosity
$\nu$	Kinematic viscosity

$i$	Type of fluid
$w$	Surface
$\infty$	Free stream
1	Upper fluid
2	Lower fluid

# **CHAPTER ONE**

## **INTRODUCTION**

### **1.1 INTRODUCTION**

In this chapter, the overview of this thesis is presented. The background of the study is discussed in section 1.2. The statement of problem and research objectives are explained in sections 1.3 and 1.4 respectively. Following by scope of research as well as the significance of the study in sections 1.5 and 1.6. Meanwhile, section 1.7 presents the research methodology. Lastly, the thesis organization is explained in section 1.8.

### **1.2 BACKGROUND OF STUDY**

Fluid is among the most important element in our world. The significance of fluid in life can be seen such as air, water, blood, oil, cloud, smoke and etc. Almost everything in this world relates to the fluid. Throughout history, it can be seen that when humans recognize fluid we can do much more to our civilization. Scientists began to develop an interest to investigate more about fluid and later define fluid as a liquid or gases that react toward a shear force (Pritchard & Leylegian, 2011). Besides, fluid consists of the substance that deforms and change according to the shape (Kambe, 2007). An area of the field that studies fluids at rest or in motion is called fluid mechanics which has been applied to the human body, dam systems, automobile industry, large scale wind turbines, energy generator and many more (Pritchard & Leylegian, 2011).

Fluid becomes interesting when it flows on a surface or toward something. The phenomena of fluid flow can be understood as a mass motion involving continuous

deformation (Kambe, 2007). Examples of fluid flow are wind, river, blood flow, respiratory air, sea current, solar wind etc. In fluid mechanics, the fluid flow can be categorized into several types. First is internal versus external flow that depends on the boundaries of the fluid flow. Second, compressible versus incompressible flow where the flow is determined by the condition of fluid's density. Third is laminar versus turbulent flow that indicated by how smooth or chaotic the flow is. The fourth is whether the fluid flow is by natural causes or by force. Another important type of flow is steady versus unsteady. This flow is known as steady if there is no change at a point with time. Otherwise, it is called unsteady. All the analysis of fluid will be based on the conservation of mass, Newton's second law of motion, the principle of angular momentum, the first law of thermodynamics and the second law of thermodynamics.

One of the important characteristics of fluid is the viscosity which can be understood as the rate of deformation of fluid. Viscosity also can be used to determine the internal stickiness of the fluid. This attribute becomes the indicator toward a classification of fluid into Newtonian and non-Newtonian fluids. The Newtonian fluid is a fluid that obeys Newton's law of viscosity where the shear stress is directly proportional to the rate of deformation and is governed by the Navier – Stokes equation. On the contrary, the relationship between the rate of deformation and shear stress for non-Newtonian fluid is not linear thus fluid does not obey Newtonian's law of viscosity. For example, honey, blood, molten plastic, liquid crystal and polymeric liquids are a part of the non-Newtonian fluid that we are dealing with in everyday life. In research, non-Newtonian fluid is categorized as Casson fluid, Jeffery fluid, nanofluid, viscoelastic fluid, micropolar fluid and many more. The rheology of non-Newtonian fluid is very interesting that leads numerous researches continue to investigate it.

In this thesis, the type of non-Newtonian fluid considered is the micropolar fluid which be a subclass of microfluid. The micropolar fluid concept initiated by Eringen (1965) which become attention to the researchers for the past few decades due to the many applications to the industry such as paints, blood, body fluid, polymers, colloidal fluids and suspension fluid (Ishak et al., 2007). Very recently, micropolar fluid is also used as a model for sliding hiatus hernia (Chandra & Kumar Pandey, 2018) and stenosed artery (Haghighi, et al., 2019). This theory explained that micropolar fluid displays microrotational effect and microrotational inertia. Besides, it can support couple stress and body couple. In order to solve the problem related to micropolar fluid, an extra equation named as angular momentum equation must be added in the governing equation. More details about the mechanics of micropolar fluid can be referred in Ariman et al. (1973), Ariman et al. (1974), Willson (1970), Lukaszewicz (1999) and Eremeyev et al. (2013).

Flows over stretched surfaces have various engineering and industrial applications like the extrusion of plastic sheets, extraction of polymer, glass blowing, drawing of wires, paper production and rubber sheets. It is important to know the flow properties of the ambient fluid, speed of collection and the rate of heat transfer at the stretching surface since the production of a quality product depends on it. The research regarding fluid flow over the moving surface was initiated by Sakiadis (1961) and Crane (1970) extended to linearly stretched plate. Many other researchers begin their interest in this topic such as Sankara and Watson (1985) as well as Na and Pop (1997). More fundamental and important facts about the stretching surface can be found in a book written by Mehmood (2017).

In addition, according to Gupta (2014), heat convection can be divided into free convection and forced convection. The convective effect present when the