

HYBRID REPAIR OF CRACKED PLATES
STRENGTHENED WITH COMPOSITE PATCHES AND
PIEZOELECTRIC ACTUATORS

BY

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ABSTRACT

Active repair of a damaged structure using piezoelectric (PZT) actuators in controlling crack by electro-mechanical effect has played a significant role in recent years. Similarly, passive repair of damaged structures by means of various composite material patches have been widely studied by many researchers during the last four decades. This thesis proposes a hybrid repair of edge-cracked and center-cracked plates by using PZT actuators at the front of the plate, and a composite patch at the back of the plate via analytical, numerical, and experimental investigation. The models relate the Mode-I stress intensity factor (SIF), composite patch, and PZT actuator parameters for an edge-cracked and center-cracked aluminium plates, respectively. The electromechanical models are based on Linear Elastic Fracture Mechanics (LEFM), the singular stress at the crack tip, and the coupling effects of the PZT actuators. The first part of this thesis presents the finite element (FE) modelling and analysis of the present model and its validation with the existing results. In the second part, two types of analytical models were presented. In the first method, the solution was obtained from Rose's equations for the cracked plate integrated with a composite patch and passive PZT actuators under uniform uniaxial load. Then, the SIF for a cracked plate due to stress produced by PZT actuators was analytically modeled using the weighted functions method. The superposition principle method (SPM) was then used to superimpose the aforementioned solutions to yield the PZT actuators and composite patch hybrid SIF. In the second method, the analytical approach was driven from the parallel mean function (PMF), which is the combination of two separate solutions of passive and active analytical methods. Both proposed analytical models' results were verified against the finite element ones. The results demonstrated relatively low errors of less than 10% between the analytical and the FE values in all the cases studied in this work. Thus, the solutions obtained using the analytical approach are acceptable for the computation of SIF with reasonable accuracy. In the third part, an experimental investigation was carried out to verify the analytical and the FE results under Mode-I loading condition of an edge-cracked plate. Additionally, a parametric analysis was conducted to understand the influence of composite patch and PZT actuators on the mitigation of the SIF. In the fourth part of this thesis, the design of experiments (DOE) method was used to optimize the process parameters that lead to minimal SIF. Therefore, three parameters were used to optimize the reduction of SIF for the case of active and passive repair, whereas four parameters were used to investigate the optimal result of the hybrid repair. The present results demonstrated that the maximum reduction of SIF is accomplished by the application of the thick composite patch with thin adhesive bond coupled with thin actuators at higher voltage. In summary, this thesis investigated the possibility and pragmatism of the hybrid repair of edge-cracked and center-cracked plates under Mode-I loading condition with analytical, FE, experimental and optimization studies.

خلاصة البحث

لقد لعب الإصلاح النشط للهيكال التالف باستخدام المشغلات الكهرو ضغطية (PZT) في التحكم في الشرخ بالتأثير الكهروميكانيكي دورًا مهمًا في السنوات الأخيرة. وبالمثل، تمت دراسة الإصلاح السليبي للهياكل التالفة عن طريق مختلف بقع المواد المركبة على نطاق واسع من قبل العديد من الباحثين خلال العقود الأربعة الماضية. تقترح هذه الأطروحة الإصلاح الهجين للألواح المشققة في الحواف والمشققة في الوسط باستخدام مشغلات PZT في مقدمة اللوحة، أما في الجزء الخلفي من اللوحة فيستخدم المواد المركبة للربط. وقد تم استخدام الطرق التحليلية، العددية والتجريبية. ترتبط النماذج بعامل شدة الإجهاد (SIF) Mode-I، والرابط ذو المواد لمركبه، ومعاملات مشغل PZT لألواح الألمنيوم المشققة بالحافة والمشققة في المركز، على التوالي. تعتمد النماذج الكهروميكانيكية على ميكانيكا الكسر المرن الخطي (LEFM)، والضغط المفرد عند طرف الشق، وتأثيرات اقتران مشغلات PZT. يقدم الجزء الأول من هذه الأطروحة نمذجة العناصر المحدودة (FE) وتحليل النموذج الحالي والتحقق من صحة الطريقة مع النتيجة الحالية. في الجزء الثاني تم تقديم نوعين من النماذج التحليلية. في الطريقة الأولى، تم الحصول على الحل من معادلات روز للوحة المكسورة المدججة مع الرقعة ذو المواد المركبة ومشغلات PZT سلبية تحت حمل أحادي المحور. بعد ذلك، تم تصميم نموذج SIF للوحة المكسورة بسبب الإجهاد الناتج عن مشغلات PZT بشكل تحليلي باستخدام طريقة الوظائف الموزونة. ثم تم استخدام طريقة مبدأ التراكب (SPM) لتركيب الحلول المذكورة أعلاه لإنتاج مشغلات PZT و SIF الهجين المركب. في الطريقة الثانية، كان النهج التحليلي مدفوعًا من دالة المتوسط المتوازي (PMF)، وهي مزيج من حلين منفصلين من الأساليب التحليلية السلبية والنشطة. أخيرًا، تم التحقق من الطريقة التحليلية المقترحة باستخدام تحليل العناصر المحدودة (FEA). أظهرت النتائج أخطاء منخفضة نسبيًا أقل من 10٪ بين القيم التحليلية وقيم FE في جميع الحالات التي تمت دراستها في هذا العمل. وبالتالي، فإن الحلول التي تم الحصول عليها باستخدام النهج التحليلي مقبولة لحساب SIF بدقة معقولة. في الجزء الثالث، تم إجراء التحقيق التجريبي للتحقق من الأسلوب التحليلي وطريقة FE في ظل حالة تحميل الوضع الأول للوحة مشققة الحافة. بالإضافة إلى ذلك، تم إجراء التحليل المعياري لفهم تأثير التصحيح المركب ومشغلات PZT على التخفيف من SIF. في الجزء الرابع من هذه الرسالة، تم استخدام طريقة تصميم التجارب (DOE) لدراسة تأثير المعلمات بدقة. لذلك، تم استخدام ثلاث معاملات لتحسين النتائج الدنيا مع قيمة مناسبة معقولة في تقليل SIF لحالة الإصلاح النشط والسليبي، بينما تم استخدام أربع معاملات للتحقق من النتيجة المثلى للإصلاح الهجين. أظهرت النتائج الحالية أن الحد الأقصى من SIF يتم تحقيقه من خلال تطبيق الرقعة ذو المواد المركبة السميكة برابطة رقيقة لاصقة مقترنة بمشغلات رقيقة عند الجهد العالي. باختصار، بحثت هذه الأطروحة في إمكانية وعملية الإصلاح الهجين للألواح المشققة بالحافة والمشققة في المنتصف في ظل حالة التحميل Mode-I مع الدراسات التحليلية، FE، التجريبية والتحسين.

APPROVAL PAGE

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DECLARATION

I hereby declare that this thesis is the result of my own investigations, except where otherwise stated. I also declare that it has not been previously or concurrently submitted as a whole for any other degrees at IIUM or other institutions.

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To Almighty Allah for means; and to my beloved parents and family, for their endless love, support and encouragement

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In the Name of Allah, the Most Compassionate, the Most Merciful

Allah - beginning with the name of - the Most Gracious, the Most Merciful Most Auspicious is he in whose control is the entire kingship; and he is able to do all things [67:1]. All Praise to Allah, the Lord of the creation, and countless blessings and peace upon our Master Mohammed, the leader of the Prophets.

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LIST OF SYMBOLS

a	Crack Length
K_I	Stress Intensity Factor
$K_{I(\text{piezo})}$	Stress Intensity Factor due to Piezoelectric Actuator
$K_{I(\text{total})}$	Total Stress Intensity Factor
K_a	Active Repair Stress Intensity Factor
K_p	Passive Repair Stress Intensity Factor
K_h	Hybrid Repair Stress Intensity Factor
σ or σ_0	Applied Tensile Load/Stress
σ_{pz} or σ_{piezo}	Piezoelectric Actuator Load
V	Applied Piezoelectric Voltage
H	Cracked Plate Hight
T	Cracked Plate Thickness
W	Cracked Plate Width
h_p	Composite Patch Hight
t_p	Composite Patch Thickness
w_p	Composite Patch Width
h_a	Piezoelectric Actuator Hight
t_a	Piezoelectric Actuator Thickness
w_a	Piezoelectric Actuator Width
h_A	Adhesive Hight

t_A	Adhesive Thickness
w_A	Adhesive Width
E	Cracked Plate Young's Modulus
ν	Cracked Plate Poison's ratio
G	Cracked Plate Shear Modulus
S_1	Mechanical strain
D_3	Electric Displacement
E_3	Electric Field
d_{31}	Piezoelectric Coefficient
S	Stiffness Ratio
Λ	Piezoelectric strain
T	Distribute Electrodes width
ϵ_{33}^T	Dielectric Constant at zero Stress
s_{11}^E	Mechanical Complaisance at zero Electric Field
S_{piezo}	Piezoelectric Actuator Force
u_y	Crack Displacement at y-direction
Γ	Crack Perimeter

LIST OF ABBREVIATIONS

2D	Two-dimensional
3D	Three-dimensional
ASTM	American Society for Testing and Materials
Coef	Coefficient
DOE	Design of Experiments
DOF	Degree of Freedom
EDM	Electric Discharge Machine
ERR	Energy Relies Rate
FE	Finite Element
FEA	Finite Element Analysis
FEM	Finite Element Method
F-Value	F-statistic, the ratio of two mean squares that forms the basis of a hypothesis test
LEFM	Linear Elastic Factor Mechanics
MS	Mean Square
NSIF	Normalized Stress Intensity Factor
NE	Number of Elements
NN	Number of Nodes
SE	Stander Error
SIF or K	Stress Intensity Factor
SS	Sum of Square

SPM	Superposition Principle Method
PM	Parallel Mean
PMF	Parallel Mean Function
PZT	Piezoelectric Actuator
P-value	p-value, the probability of obtaining an F statistic at least as extreme as that observed when the null hypothesis is true
VIF	Variance Inflation Factor

CHAPTER ONE

INTRODUCTION

1.1 OVERVIEW

This chapter covers the introduction, problem statement, research philosophy, scope, methodology, and objectives of the current research. First, a presentation, including a brief overview of the topic, is presented, and followed by the research problem and reasoning to the solution based on the work's philosophy. At this point, the scope of the research is briefly clarified. With a specific end goal to answer the problem statement, objectives are laid down. Then, the critical flow of the current study is explained in the research methodology. Finally, the outline of the thesis is presented.

1.2 BACKGROUND OF THE STUDY

Aircraft nowadays are maintained in such a way that safety is given the highest priority. To implement that, aircraft structures that may lead to crack propagation, damages, and fractures are to be replaced with new structures. Damages such as delamination, notch, and crack in various fields of engineering are inevitable especially in the aerospace field, and these damages are mostly due to fatigue, corrosion, and accidents. To study the cracked/damaged structures performance, fracture mechanics techniques will be useful in such a problem. Fundamentally, fracture mechanics is the study of mechanical behaviour in cracked materials subjected to an applied load. Crack is essentially a fracture occurring at the interface of two adjacent layers. The mechanical behaviour of a solid containing a crack of a specific geometry and size can be predicted by evaluating the stress intensity factor (SIF) for K_I , K_{II} , and K_{III} (Perez, 2004). Therefore, many studies have been carried out

to investigate the stress intensity factor of the repaired plate in either passive or active structural repair.

From the last two decades, more attention was directed towards the applications of smart materials in engineering structures. These materials possess attributes that can be altered desirably under a controlled environment through temperature, stress, and electric or magnetic field, which act as external stimuli. The most typical examples of such materials, widely employed in different areas, are shape memory alloys and piezoelectric materials. The latter of the two is characterized by a trait referred to as the electromechanical effect. This trait is because of interaction between the electrical and mechanical properties of a given material. When mechanical stress is applied to a piezoelectric material, an electric field is produced (direct piezoelectric effect), and, conversely, mechanical deformation will be generated when an electric field is applied (converse piezoelectric effect). (IEEE, Standard on Piezoelectricity 1988). On such an application, few studies have been highlighted in this section which used the PZT actuator for repair performance.

Wang et al., (2010) studied the repair of cracked structure to give its original functionality by enhancing the smart materials and evaluated the SIF. Abuzaid et al., (2015a, b) proved that to recover its original shape hampered by crack, SIF is the most crucial parameter to be found. The most widely used smart structures are shape memory alloy and piezoelectric (PZT) material. PZT materials are characterized by an electromechanical effect; this characteristic is because of the interaction between electrical and mechanical properties of a given material. The advantage of using a PZT actuator is to reduce the maintenance inventories and increase the life cycle of the cracked structure. The effect of the adhesive bond on an active repair is to transmit the induced stresses by the PZT actuator to the cracked structure, and this effect will

reduce the SIF. Platz et al., (2011) concluded that the PZT actuator is also applicable for actively reducing the crack propagation with a lower cyclic SIF near the crack tip by actively inducing mechanical compression forces.

Over the four decades, passive repair has been extensively studied using a bonded composite patch which can be seen in some cases in this section. Benyahia et al., (2014) stated that the composite patches are widely used in many engineering applications, and different types of composite patches were used for the passive repair of isotropic materials. However, the composite patch is commonly used in the cracked area. Frostig (2006) studied that the composite patches are available in many types based on the property of the material used and depending on the loading condition and modes. Compression stresses in bonded strips tend to buckle, trigger, and accelerate the unstable growth of crack to the unbonded region of the fractured plate. It is essential to know the patch dimensions since the higher thickness of the patch will result in a high reduction in SIF; on the contrary, high thickness will result in higher weight. Hence, it is desired to keep the patch and adhesive thickness to the lowest. The most common composite materials used for the repair of cracked material are boron/epoxy, glass/epoxy, graphite/epoxy (Mhamdia et al., 2011), E-glass/epoxy (Bouiadjra et al., 2010), and carbon/epoxy (Ricci et al., 2011). Composite structures have been increasingly used in load-bearing structures because of their excellent strength-to-weight and stiffness-to-weight characteristics, compared to many conventional materials and metallic alloys (Gibson, 2011).

However, more recently, researchers have been exploring active repair and stress control of mechanical structures including the piezoelectric actuator electromechanical behaviour (Wang and Wu, 2012). The converse effect of the piezoelectric actuator has made it possible to induce control moments and forces on