

FAT-BASED ADAPTIVE CONTROL (FATAC) OF
COOPERATIVE MANIPULATORS FOR HANDLING A
DEFORMABLE OBJECT

BY

ABDUL RAHMAN BIN SAMEWOI

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International Islamic University Malaysia

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ABSTRACT

Handling a flexible object by cooperative manipulators is more complicated than handling the rigid one as it involves the vibration of the object. Since the vibration has been known as the capacity for disturbance, discomfort, damage, and destruction, it needs to be suppressed. The system consists of two cooperative manipulators handling a flexible beam that is modelled in Partial Differential Equation (PDE) form and employed the singular perturbation method to produce slow and fast subsystems. Despite the advantages offered by the PDE-based system, less work has been conducted in designing a controller for handling deformable objects by cooperative manipulators based on the PDE-based model and considering the model uncertainties. This study proposes a composite controller that comprises Function Approximation Technique (FAT)-based Adaptive Controller (FATAC) for the slow subsystem to control two cooperative manipulators in handling the deformable object under uncertain model parameters and Velocity Feedback Controller (VFC) for the fast subsystem to suppress the vibration of the deformable object. Stability analysis has been carried out for each subsystem to satisfy Tikhonov's Theorem. Simulation tests have been carried out to measure the performance of designed controllers. For the slow subsystem, the simulation results showed that the root-mean-square (RMS) tracking error of the beam's midpoint are 0.004599 m for X-position, 0.001697 m for Y-position, and 0.005186 rad. for the orientation under the proposed FATAC. For the fast subsystem, the simulation results proved that the proposed VFC has successfully worked well as the transverse vibration of the beam is completely suppressed within 0.8 s. Hardware experimental tests have also been carried out to validate the proposed controller. For slow subsystem, the coding of proposed FATAC is developed to control two cooperative manipulators so that the positions of the beam's midpoint track the circular desired trajectory. The experimental results showed that the position tracking of the beam's midpoint which is controlled by two cooperative manipulators under the proposed FATAC has been successfully achieved with the RMS tracking error of 0.914 cm and 1.126 cm for X and Y-directions, respectively. For the fast subsystem, the calibration of the strain gauge sensor has been made for the preparation to design VFC. The ultimate stage in the fast subsystem is validating the VFC by experimental hardware test to suppress the vibration of the flexible beam. The experimental results proved that the proposed VFC for the fast subsystem has successfully suppressed the beam vibration while moving the flexible beam according to the desired trajectory. The simulation and hardware experiment results verified that the proposed composite controller comprises FATAC for the slow subsystem that has successfully driven cooperative manipulators to handle the deformable object to follow the desired trajectories and VFC for the fast subsystem that has successfully suppressed the transverse vibration of the deformable object.

خلاصة البحث

التعامل مع الجسم من قبل المناور التعاوني هو أكثر تعقيدا من العامل مع الاجسام الجامده لانه ينطوي على اهتزازات. منذ ان كان الاهتزاز يعرف باسم القدره على الاضطراب وعدم الراحة والضرر والدمار فإنه يحتاج الى قمعها. يتكون النظام من اثنين مناورين متعاونين والذين يتعاملون مع شعاع مرن تم تصميمه على غرار المعادله التفاضلية الجزئية (PDE) ويستخدم طريقة الاضطراب المفرد لإنتاج انظمه فرعية بطيئه وسريعه. وعلى الرغم من المزايا التي يوفرها النظام القائم على PDE, فقد تم اجراء قدر اقل من العمل في تصميم وحده تحكم للتعامل مع الاشياء القابله للتشوه من قبل المناورين التعاونيين على اساس النموذج القائم على PDE والنظر في عدم اليقين في النموذج. وتقترح هذه الدراسه وحده تحكم مركبة تضم وحدة تكيفية تعتمد على تقنية تقريب الوظائف (FATAC) FAT للنظام الفرعي البطئ للتحكم في اثنين من المناورين التعاونيين في التعامل مع الكائن القابل للتشوه تحت معلمات النموذج غير المؤكدة ووحدة التحكم في سرعة التغذية المرتدة (VFC) للنظام الفرعي السريع لقمع اهتزاز الكائن القابل للتشوه. وقد تم اجراء تحليل الاستقرار لكل نظام فرعي لإرضاء تيجونوف. وتم إجراء اختبارات المحاكاة لقياس اداء وحدات التحكم المصممه. وبالنسبه للنظام الفرعي البطئ فقد اظهرت نتائج المحاكاه ان خطأ تتبع الجذر المتوسط (RMS) لنقطة منتصف الحزمة هو $X 0.004599$ وايضا $Y 0.001697$ وايضا $rad 0.005186$ للتوجيه تحت FATAC المقترحة. وايضا للنظام الفرعي السريع اثبتت النتائج ان VFC المقترح قد نجح في العمل بشكل جيد حيث يتم قمع الاهتزاز العرضي للحزمه تماما في غضون. وللنظام الفرعي البطئ تم تطوير ترميز FATAC المقترح للتحكم في اثنين من المناورين التعاونيين بحيث تتبع مواقع نقطة منتصف الحزمه المسار الدائري المطلوب. واطهرت النتائج التجريبيه ان تتبع موضع نقطه منتصف الحزمه التي يتم التحكم فيها بواسطه مناورين متعاونين بموجب FATAC المقترح قد تحقق بنجاح مع خطأ تتبع RMS البالغ $cm 0.914$, وايضا $cm 1.126$ للمحورين X-Y على التوالي. بالنسبه للنظام الفرعي السريع فقد تم اجراء معاييره مستشعر قياس الضغط للتحضير لتصميم VFC وفي المرحله النهائية للنظام الفرعي السريع وهي التحقق من صحه VFC عن طريق اختبار الاجهزه التجريبي لوقف اهتزاز الحزمه المرنة واثبتت النتائج التجريبيه ان VFC المقترح للنظام الفرعي السريع فقد نجح في وقف اهتزاز الحزمه اثناء تحريك الحزمه المرنة وفقا للمسار المطلوب. واخيرا فقد تحققت نتائج تجريره المحاكاه والاجهزه من ان وحدة التحكم المركبه المقترحة تضم FATAC للنظام الفرعي البطئ الذي نجح في دفع المناورين

التعاونيين للتعامل مع الجسم القابل للتشوه لمتابعه المسارات المطلوبه واستخدام نظام VFC للنظام الفرعي السريع الذي نجح في وقف الاهتزاز العرضي للجسم القابل للتشوه.

APPROVAL PAGE

I certify that I have supervised and read this study and that in my opinion, it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a thesis for the degree of Master of Science (Mechatronics Engineering).

.....
Norsinnira Zainul Azlan
Supervisor

.....
Md. Raisuddin Khan
Co-Supervisor

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a thesis for the degree of Master of Science (Mechatronics Engineering).

.....
Hasmawati Antong
Internal Examiner

.....
Ruhizan Liza Ahmad Shauri
External Examiner

This thesis was submitted to the Department of Mechatronics Engineering and is accepted as a fulfilment of the requirement for the degree of Master of Science (Mechatronics Engineering).

.....
Syamsul Bahrin Abdul Hamid
Head, Department of
Mechatronics Engineering

This thesis was submitted to the Kulliyah of Engineering and is accepted as a fulfilment of the requirement for the degree of Master of Science (Mechatronics Engineering).

.....
Sany Izan Ihsan
Dean, Kulliyah of Engineering

DECLARATION

I hereby declare that this thesis is the result of my own investigations, except where otherwise stated. I also declare that it has not been previously or concurrently submitted as a whole for any other degrees at IIUM or other institutions.

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LIST OF ABBREVIATIONS

AMM	Assumed Mode Method
CTC	Computed Torque Control
DAQ	Data Acquisition
DC	Direct Current
DOF	Degree of Freedom
FAT	Function Approximation Technique
FATAC	Function Approximation Technique-based Adaptive Controller
FEM	Finite Element Method
MFRAC	Model-Free Robust Adaptive Control
MOSFET	Metal Oxide Semiconductor Field Effect Transistor
PD	Proportional-Derivative
PDE	Partial Differential Equation
PWM	Pulse Width Modulation
RMSE	Root Mean Square Error
SMC	Sliding Mode Control
SMC-FAT	Sliding Mode Control - Function Approximation Technique
VFC	Velocity Feedback Control

LIST OF SYMBOLS

$q_{ij}, \dot{q}_{ij}, \ddot{q}_{ij}$	Joint displacement, velocity, and acceleration of j -th link for i -th manipulator, respectively
l_{ij}	Link lengths of j -th link for i -th manipulator
x_{ei}	x -position of the end-effectors for the i -th manipulator
y_{ei}	y -position of the end-effectors for the i -th manipulator
x_{wi}	x -position of the wrist for the i -th manipulator
y_{wi}	y -position of the wrist for the i -th manipulator
ϕ_i	The orientation of the wrists for the i -th manipulator
$q_i, \dot{q}_i, \ddot{q}_i$	3x1 vectors of generalised joint displacements, velocities and accelerations for the i -th manipulator, respectively
$M_i(q_i)$	3x3 symmetric positive definite inertia matrix for the i -th manipulator
$C_i(q_i, \dot{q}_i)$	3x3 Coriolis and Centrifugal matrix for the i -th manipulator
$G_i(q_i)$	3x1 vector of gravitational components i -th manipulator
$J_i(q_i)$	3x1 Jacobian matrix of i -th manipulator
τ_i	3x1 vector of input torque applied at each joint of i -th manipulator
f_i	3x1 vector of the interaction force between i -th manipulator and the flexible beam
m_{ij}	Mass of the i -th links of the j -th manipulator

l_{cij}	Distance from the centre of mass of the i -th links of the j -th manipulator
I_{ij}	Moment of inertia of the i -th links of the j -th manipulator
q, \dot{q}, \ddot{q}	6x1 vectors of generalised joint displacements, velocities and accelerations of two manipulators, respectively
M_m	6x6 symmetric positive definite inertia matrix of two manipulators
C_m	6x6 Coriolis and Centrifugal matrix of two manipulators
G_m	6x1 vector of gravitational components of two manipulators
J	6x6 Jacobian matrix of two cooperative manipulators
τ	3x1 vector of input torque applied at each joint of two manipulators
f	6x1 vector of interaction force and moment between two manipulators and the flexible beam
L	Length of the flexible beam
ρ	The density of the flexible beam
m	Mass of the flexible beam
X_r, Y_r -frame	Fixed coordinate frame
xy -frame	Moving coordinate frame
$X_{mp}, \dot{X}_{mp}, \ddot{X}_{mp}$	3x1 vectors of positions/orientation, velocity, and acceleration, respectively, of the flexible beam's midpoint
x_o	x -position of the flexible beam's midpoint
x_{od}	desired x -position of the flexible beam's midpoint

y_o	y-position of the flexible beam's midpoint
y_{od}	Desired y-position of the flexible beam's midpoint
θ	The orientation of the flexible beam's midpoint
θ_d	Desired orientation of the flexible beam's midpoint
F_{1x}, F_{1y}	The forces applied by the first manipulator at the first end of the flexible beam
F_{2x}, F_{2y}	The forces applied by the second manipulator at the second end of the flexible beam
t	Slow time scale
t_0	Slow initial time
x	Spatial coordinate
$\eta(x, t)$	Transverse displacement (i.e. flexible parameter) that varies with x and t
$\ddot{\eta}$	The second derivative of $\eta(x, t)$ with respect to t
η^{iv}	The fourth derivative of $\eta(x, t)$ with respect to x
R_{rf}	6x3 matrix of velocity relations between the beam's midpoint and both ends of the beam
δU	Variation of potential energy, U
δT	Variation of kinetic energy, T
δW	Variation of work done due to external forces, W
t_1 and t_2	Any two instances of time with $t_2 > t_1 > 0$.
M_{bfr}	3x3 inertia matrix of the beam with flexible and rigid parameters

C_{brf}	3x1 centrifugal vector of the beam with flexible and rigid parameters
η_{brf}	3x1 vibration vector of the beam with flexible parameters only
G_{brf}	3x1 gravitational vector of the beam with flexible and rigid parameters
F_{brf}	3x6 force transformation matrix with flexible and rigid parameters
F_{ff}	1x6 force transformation matrix in the transverse vibration
E	Moment of inertia of the beam (kgm^2)
I	Young's modulus of the beam (Pa)
$v(x,t)$	New introduced variable
v_s	Slow variable
$v_f, \hat{v}_f, \hat{\hat{v}}_f$	Fast variable, its first and second derivatives, respectively, with respect to the fast time scale, h
C	A dimensionless parameter which has a large value for the different materials
μ	Perturbed parameter
U_{cr}	Overall cooperative manipulators input
h	Fast time scale
A	The differential operator in Hilbert space
f_f	The interaction forces between a manipulator and the flexible beam
U_c	Composite controller

U_s	Slow subsystem controller
U_f	Fast subsystem controller
$X_{mpd}, \dot{X}_{mpd}, \ddot{X}_{mpd}$	3x1 vectors of desired positions/orientation, velocity, and acceleration, respectively, of the flexible beam's midpoint
e, \dot{e}, \ddot{e}	Tracking error
U_{pd}	Control input function
K_v and K_p	PD gains
$\Delta_c(s)$	Closed-loop characteristics polynomial
F_{ff}^\dagger	Pseudo-inverse of F_{ff}
Π	The operator that is neither self-adjoint nor positive definite
k	Positive gain
Q	Bounded and positive definite operator
A	Positive-definite operator
Λ	$diag(\lambda_1, \lambda_2, \dots, \lambda_n)$ and $\lambda_n > 0$ for $i = 1, \dots, n$.
K_d	Positive-definite matrix
$\hat{M}_{cr}, \hat{C}_{cr}$ and \hat{G}_{cr}	Approximation of M_{cr}, C_{cr} and G_{cr} , respectively
v and \dot{v}	Signal vector and its first derivative, respectively
W_M, W_C and W_G	Weighting matrices
Z_M, Z_C and Z_G	Matrices of basis function
$\beta_{(\cdot)}$	Number of basis functions
\tilde{W}_M, \tilde{W}_C and \tilde{W}_G	Approximation error

$V(s, \tilde{W}_M, \tilde{W}_C, \tilde{W}_G)$	Candidate of Lyapunov-like function
Q_M, Q_C and Q_G	Positive-definite weighting matrices
$\dot{\hat{W}}_M, \dot{\hat{W}}_C, \dot{\hat{W}}_G$	Update laws
r_x and r_y	Values that determine the radius of the desired circular trajectory
c_x and c_y	Values that determine the centre point of the desired circular trajectory